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DYNAMIC GROWTH MODEL FOR BIRCH STANDS IN NORTHWESTERN SPAIN

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Abstract

A dynamic whole-stand growth model for birch (*Betula pubescens* Ehrh.) stands in Northwestern Spain is presented. In this model, the initial stand conditions at any point in time are defined by three state variables (number of trees per hectare, stand basal area and dominant height), and are used to estimate total or merchantable stand volume for a given projection age. The model uses three transition functions expressed as algebraic difference equations to project the corresponding stand state variables at any particular time. In addition, the model incorporates a function for predicting initial stand basal area, which can be used to establish the starting point for the simulation. Once the state variables are known for a specific moment, a distribution function is used to estimate the number of trees in each diameter class by recovering the parameters of the Weibull function, using the moments of first and second order of the distribution. By using a generalized height diameter function to estimate the height of the average tree in each diameter class, combined with a taper function that uses the above predicted diameter and height, it is then possible to estimate total or merchantable stand volume.

Key words: *Betula pubescens* Ehrh., even-aged stands, whole-stand growth model, generalized algebraic difference approach, basal area disaggregation, Galicia.

Introduction

The *Betula* genus is distributed throughout most of Europe, where it is mainly represented by two stand-forming tree species, *Betula pubescens* Ehrh. and *Betula pendula* Roth. In the Iberian Peninsula, these two taxa are freely hybridized, and their taxonomy is somewhat confused, with each possessing a number of different nomenclatural identities (Castroviejo et al. 1990). Of the two taxa, *Betula pubescens* is the more oceanic, westerly distributed taxon, while *Betula pendula* is rarer and occupies more easterly and southerly, high altitude, mesic locations (Stevenson 2000). In Galicia (north-western Spain) Downy birch (*Betula pubescens*) grows at 0–1700 m above see level, although it is more abundant in the north-eastern area of this region at altitudes above

400–500 m. This species requires high moisture all year round and may be considered a fast-growing pioneer tree, which readily colonises open ground originated by human activity (burning, cutting or grazing) or natural disturbances. In Galicia there are currently 32,000 ha of stands that include birch as the main tree species (Xunta de Galicia 2001). However, it is much less common than it could be in Galicia as an integral part of the climax vegetation in the area, as a potentially useful species for colonising part of the approximately 635,000 ha (almost one third of the forest area in Galicia) that is at present unproductive or is colonised by scrub. Despite this, there is a notorious lack of reference information regarding sivilculture, growth and yield of this species.

Considering that growth and yield models are of primary concern in making forest management decisions, the objective of this study was to develop a management-oriented dynamic whole-stand model for simulating the growth of even-aged birch stands in Galicia.

Material and Methods

The data used to develop the model were obtained from three different sources. Initially, in the winter of 1998–1999 a network of 137 permanent plots was established in even-aged, birch-dominated stands (85% or more of the standing basal area consisting of birch). The plots were located throughout the area of distribution of this species in Galicia, and were subjectively selected to represent the existing range of ages, stand densities and sites. The plot size ranged from 200 m^2 to 1000 m^2, depending on stand density, in order to achieve a minimum of 30 trees per plot. All the trees in each sample plot were labelled with a number. The diameter at breast height (1.3 m above ground level) of each tree was measured with calliper twice (perpendicular to each other) to the nearest 0.1 cm and the arithmetic mean of the two measurements was calculated. Total height was measured to the nearest 0.5 m with a hypsometer in a randomized sample of 30 trees, and in an additional sample including the dominant trees. Descriptive variables of each tree were also recorded, e.g., if they were alive or dead.

Taking into account that some plots had disappeared because of forest fires or clear-cutting, a subset of 54 of the initially established plots was re-measured in the winter of 2008-2009. These plots were selected for some of the dynamic components of the model. The interval between the measurements was considered sufficient to absorb the short-term effects of abnormal climatic extremes (von Gadow and Hui 1999). The first two sources of data were the two inventories carried out in 1998–1999 and 2008–2009.

Apart from these inventories, where possible, two undamaged dominant trees were destructively sampled in the winter of 1998–1999, constituting a final sample of 214 trees. They were selected as the first two trees found outside the plots but in the same stands within ±5% of the mean diameter at 1.3 m above ground level and mean height of the dominant trees. The trees were felled leaving stumps of average height 0.16 m; total bole length was measured to the nearest 0.01 m. The logs were cut at approximately 1 m intervals until the diameter was 7 cm and at 2 m intervals thereafter. The number of rings was counted at each cross sectioned point and then converted to stump age. Additionally, 90 non-dominant trees were felled outside 23 locations in the winter of 2008-2009 to ensure representative distribution by diameter and height classes for taper function development. Log volumes (stem parts with merchantable size) were calculated by Smalian's formula. The top of the tree was considered as a cone. Tree volume above stump height was aggregated from the corresponding log volumes and the volume of the top of the tree. The third source of data corresponds to the 304 trees felled. Summary statistics of the stand and tree variables used in model development are shown in Table 1.

Model structure

The model is similar in structure to those developed by Diéguez-Aranda et al. (2006a) and Castedo-Dorado et al. (2007), and is based on the state-space approach (García 1994). It can be classified as a variable-density whole stand model in which stand volume is aggregated from mathematically generated diameter classes. A two-stage process that first predicts future stand density and then uses this information to estimate future stand volume allows predicting growth by subtraction (Davis et al. 2001).

Three state variables (dominant height, number of trees and basal area) define the initial stand conditions at any point in time in the model. These variables are used to estimate stand volume, classified by timber assortments, for a given projection age. Three transition

Table 1. Summarised data corresponding to the sample of plots and trees used for model development.

Variable	1st inventory (137 plots)				2nd inventory (54 plots)			
	mean	min	max	S. D.	mean	min	max	S. D.
A, years	30.3	12.0	70.0	10.1	38.9	22.0	56.0	10.0
H, m	15.3	7.2	24.4	3.6	18.3	11.0	24.5	3.0
N, trees ha^{-1}	1750	390	6000	1099	1433	350	4480	836
B, m^2 ha^{-1}	24.0	3.3	66.5	10.3	30.6	9.2	71.8	11.1
	304 trees							
d, cm	20.0	7.3	39.2	5.97				
h, m	14.5	6.2	24.4	3.42				
h_{st}, m	0.16	0.0	0.5	0.08				

Note: A = stand age; H = dominant height, defined as the mean height of the 100 largest-diameter trees per hectare; N = number of trees per hectare; B = stand basal area (only live trees were included in the calculations for N and B); d = diameter at breast height over bark; h = total tree height; h_{st} = stump height.

functions, expressed as algebraic difference equations, are used to project the corresponding stand state variables at any particular time. In addition, the model incorporates a function for predicting initial stand basal area, which can be used to establish the starting point for the simulation. A distribution function is used to estimate the number of trees in each diameter class, once the state variables are known for a specific age, by recovering the parameters of the Weibull function, using the distribution first- and second-order moments (arithmetic mean diameter and variance, respectively). Finally, it is possible to estimate total or merchantable stand volume (which depends on specified log dimensions) by using a generalized height-diameter function, to estimate the height of the average tree in each diameter class, combined with a taper function that uses the above predicted diameter and height.

The following sections describe how each of the three transition functions and the disaggregation system were developed.

Development and fitting of transition functions

The site quality equation, which combines compatible site index and dominant height growth models in one common equation, was developed by Diéguez-Aranda et al. (2006b) using data from stem analysis of the 214 dominant trees. The model was derived using the generalized algebraic difference approach (GADA, Cieszewski and Bailey 2000) on the basis of the model proposed by Cieszewski (2002). The fitting was done in one stage using the base-age-invariant dummy variables method (Cieszewski et al. 2000), expanding the error term with a second-order continuous-time autoregressive error structure to correct the inherent autocorrelation of the longitudinal data set used. This method was programmed using the SAS/EST® MODEL procedure (SAS Institute Inc. 2004a).

A dynamic equation was developed for predicting the reduction in tree number due to density-dependent mortality, which is mainly caused by competition for light, water and soil nutrients within a stand. Among the different models reported in the literature for modelling regular mortality, those included in Diéguez-Aranda et al. (2005b) were evaluated. They are equations in algebraic difference form, which were fitted to the data of the 54 plots measured twice by ordinary least squares using the SAS/STAT® NLIN procedure (SAS Institute Inc., 2004b).

The stand basal area growth of an even-aged forest depends on stand age, stand density (defined as number of trees per hectare or basal area), and site productivity (Murphy and Farrar 1988). Nevertheless, not all the equations which have been used for projecting stand basal area include these three variables. In developing the transition function for stand basal area, the equations reported in Diéguez-Aranda et al. (2005a) were evaluated. The equations were fitted to the data of the 54 plots measured twice by ordinary least squares using the SAS/STAT® NLIN procedure (SAS Institute Inc. 2004b).

Disaggregation system

The two-parameter Weibull function (Equation 1) was used to model the diameter distribution of the 137 birch plots from the first inventory and of the 54 plots re-measured:

$$f(x) = \left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1} e^{-\left(\frac{x}{b}\right)^{c}} \quad (1)$$

where x is the random variable, b the scale parameter of the function, and c the shape parameter that controls the skewness.

Several methods for parameter estimation were preliminary tested and compared for their goodness of fit. A parameter recovery method through moments and a parameter prediction method based on least squares difference estimation techniques proved best, but considering that the moments method warrants that the sum of the disaggregated basal area obtained by the Weibull function equals the stand basal area provided by an explicit growth function of this variable, the moments method was selected for application in the present study. The function parameters were recovered from the first raw moment, which is the arithmetic mean diameter $\bar{d}$, and the second central moment, which is the variance of the distribution (var), estimated by the arithmetic and the quadratic mean diameters ($\mathrm{var} = d_g^2 - \bar{d}^2$) (Diéguez-Aranda et al. 2006a), using the following expressions:

$$\mathrm{var} = \frac{\bar{d}^2}{\Gamma^2\left(1+\frac{1}{c}\right)}\left[\Gamma\left(1+\frac{2}{c}\right) - \Gamma^2\left(1+\frac{1}{c}\right)\right] \quad (2)$$

$$b = \frac{\bar{d}}{\Gamma\left(1+\frac{1}{c}\right)} \quad (3)$$

where Γ is the Gamma function.

The arithmetic mean diameter was the only variable to be modeled through a relationship on the quadratic mean diameter and other stand level variables by the function $\bar{d} = d_g - \exp(\mathbf{X\beta})$, where $\mathbf{X}$ is a vector of stand variables and $\boldsymbol{\beta}$ is a vector of parameters to be estimated. This equation was fitted to the data by ordinary least squares using the SAS/STAT® NLIN procedure (SAS Institute Inc. 2004b).

A generalized height-diameter relationship was developed to estimate the height of the average tree in each diameter class. Several models that predict the dominant height of the stand when the diameter at breast height of the subject tree equals the dominant diameter of the stand were evaluated (Crecente-Campo et al. 2010). They were fitted to the data of the 137 plots from the first inventory and of the 54 plots re-measured by ordinary least squares using the SAS/STAT® NLIN procedure (SAS Institute Inc. 2004b).

In order to obtain total- and merchantable-tree volume from the average tree-diameter of each class and its estimated height, a modification of the Kozak's (2004) variable exponent taper function was fitted to data of diameter outside bark and height of the 304 destructively sampled trees. For this purpose, a mixed-model was used and fitted using the SAS macro %NLINMIX (SAS Institute Inc. 2004b).

Selection of the best equation in each module and overall evaluation of the model

The comparison of the estimates of the different models fitted in each module was based on numerical (coefficient of determination –R^2– and root mean square error –RMSE) and graphical (plots of studentized residuals against the estimated values, and graphs of the fitted curves overlaid on the trajectories of different variables) analyses.

For the overall evaluation of the model, observed state variables from the first inventory of the 54 plots measured twice were used to estimate total stand volume at the age of the second inventory. Total stand volume was selected as the principal objective variable because it is the critical output of the whole model, its estimation involves all the functions included in it and is closely related to economical assessments. Estimations of this variable and of the state variables were evaluated in terms of the critical error (Huang et al. 2003).

Results and Discussion

Table 2 summarizes the equations selected for each sub-model. All parameter estimates were significant at a 5% level. The estimated Weibull functions modeled successfully all but 4 of the 191 examined diameter distributions, based on the Kolmogorov-Smirnov test. Figures 1–3 show the transition functions fitted curves overlaid on the observed trajectories.

It can be observed that the fitted equations follow the observed growth trajectories well, especially in the case of the stand survival function. The accuracy of this function over a wide range of ages and other stand conditions ensures that the projections of the final output variables of the whole model (e.g., stand or merchantable volume) are realistic. This equation is especially important when light thinnings are carried out (Avila and Burkhart 1992), as was the case in some of the studied stands. After heavy thinning operations it seems reasonable to assume that mortality is negligible (Castedo-Dorado et al. 2007).

As regards the stand basal area projection equation, initial basal area and initial age do not seem to provide enough information about the future trajectory of the basal area of the stand, regardless its thinning history, thus number of stems is also included as an explanatory variable. The stand basal area model has an asymptote of 74.4 m^2 ha^{-1}, which corresponds to biological expectations, at least for the stand conditions analysed in this study (Table 1). The basal area initialization equation will work well in unthinned or lightly thinned stands, although the fitting statistics showed worse results than other submodels. Because the number of trees per hectare varies over time, the initialization and the projection functions are not compatible. However, this is not a major problem because the initialization function would only be used to provide an initial value of stand basal area when no inventory data are available (Amateis et al. 1995).

Explanatory variables of the components of the disaggregation system can be easily obtained at any point in time from dominant height, number of trees and basal area transition functions. The only exception is dominant diameter of the generalized *h-d* relationship, which is a variable that is difficult to project (Lappi 1997) and must, therefore, be estimated from the diameter distribution.

Table 2. Equations selected for each sub-model and goodness of fit statistics.

	Equation	R^2	RMSE
Transition functions	**Dominant height growth/Site index**		
	$H_2 = \dfrac{19.80 + X_0}{1 + 758.0/X_0\, A_2^{-1.398}}$, with $X_0 = \dfrac{H_1 - 19.80 + \sqrt{(19.80 - H_1)^2 + 3032 H_1 t_1^{-1.398}}}{2}$	0.989	0.505 m
	Number of trees per hectare reduction		
	$N_2 = \left(N_1^{-1/1.577} + 0.01255\left(\left(\dfrac{A_2}{100}\right)^{1.577} - \left(\dfrac{A_1}{100}\right)^{1.577} \right) \right)^{-1.577}$	0.978	120 trees ha^{-1}
	Stand basal area growth		
	$G_2 = G_1^{A_1/A_2} \exp\left(2.290(1 - A_1/A_2) + 0.2521(\ln N_2 - A_1/A_2 \ln N_1)\right)$	0.950	2.50 m^2 ha^{-1}
Auxiliary relationships	**Stand basal area initialization**		
	$G = -77.14 + 0.1189A + 1.894H + 9.559 \log N$	0.760	5.44 m^2 ha^{-1}
	Disaggregation (arithmetic mean diameter)		
	$\bar{d} = d_g - \exp(-1.418 + 0.0884H - 0.0395S)$	0.929	0.27 cm
	Generalized height-diameter relationship		
	$h = 1.3 + (H - 1.3)\exp\left((-0.3834H + 0.5139N/1000)(1/d - 1/d_0)\right)$	0.778	1.78 m
	Taper equation		
	$d_i = 0.9652 d^{0.9421} h^{0.08482} x^{4.094 - 0.3939q^4 - 0.4112(1/e^{d/h}) - 4.153x^{0.1} + 2.813(1/d) + 0.05345h^w + 0.3332x}$ with $x = w/\left(1 - (1.3/H)^{1/3}\right)$, $w = 1 - q^{1/3}$, $q = h_i/h$	0.950	1.56 cm

Note: H_1, H_2, N_1, N_2, B_1, B_2 = dominant height (m, defined as the mean height of the 100 largest-diameter trees per hectare), number of trees per hectare, and stand basal area (m^2 ha^{-1}) at initial A_1 and final A_2 stand projection ages (years), respectively; log = natural logarithm; $\bar{d}$ = arithmetic mean diameter (cm); S = site index (m, at a reference age of 20 years); h = total tree height (m); d = diameter at breast height (cm, 1.3 m above ground level); d_0 = dominant diameter (cm, average value of the 100 largest-diameter trees per hectare); d_i = top diameter over bark at height h_i (cm); h_i = height above the ground to top diameter d_i (m). Total- and merchantable-tree volumes must be computed numerically. Stand volumes are aggregated from mathematically generated diameter classes.

Total stand volume was selected in the present study as the critical output variable for the whole-stand growth model, although other stand variables can be assessed on the basis of this model (e. g., biomass, carbon pools). A critical error of 20% was obtained when projecting total stand volume from the first to the second inventory; critical errors of 14–15% were obtained for dominant height, number of trees per hectare, and stand basal area. In this step, 84% of the examined diameter distributions passed the Kolmogorov-Smirnov test ($\alpha = 5\%$). Considering the required accuracy in forest growth modelling, where a mean prediction error of the observed mean at 95% confidence intervals within ±10–20% is generally realistic and reasonable as a

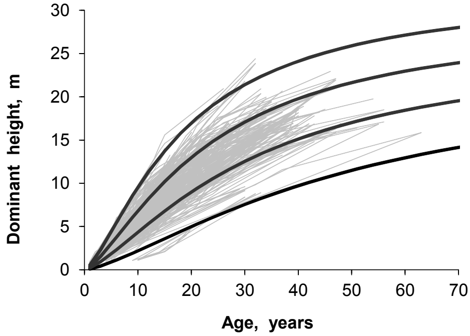

Fig. 1. Curves for site indices of 5, 9, 13 and 17 m at a reference age of 20 years overlaid on the profile plots of the data set.

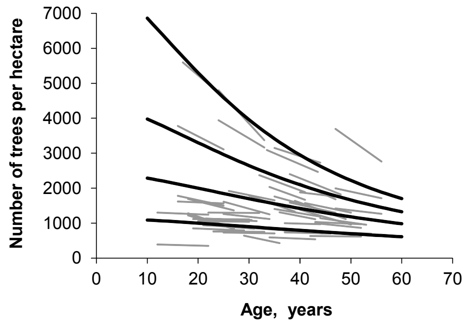

Fig. 2. Trajectories of observed and predicted tree number per hectare over time. Model projections for spacing conditions of 1000, 2000, 3300 and 5300 trees per hectare at 20 years.

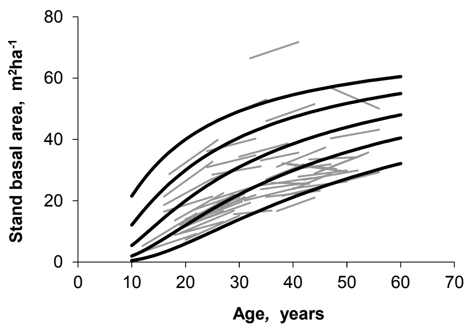

Fig. 3. Stand basal area growth curves for stand basal areas of 6, 12, 20, 30 and 40 m^2 ha^{-1} at 20 years overlaid on the trajectories of observed values over time.

limit for the actual choice of the acceptance and rejection levels (Huang et al. 2003), we can state, on the basis of the critical error statistics obtained, that the model provides satisfactory predictions.

As the model was based primarily on data from stands of ages greater than 12 years old, predictions of site index for younger stands should be made with caution, because at young ages erratic height growth may lead to erroneous site classifications. Apart from this exception, the model may be used over the expected rotation of the species in the region of study (~ 60–70 years).

The most important limitation of the model is that it does not consider the later effect of thinning and pruning before the trees fully occupy the additional space that has been made available to them. However, this effect does not seem to be important in our case since very heavy thinning treatments were not considered (García 1990).

The relatively simple structure of the stand growth model makes it suitable for embedding into landscape-level planning models and other decision support systems that enable forest managers to generate optimal management strategies. Nevertheless, because of the large number of calculations needed to obtain outputs (especially those involving use of the disaggregation system), the model will be implemented into the GesMO® 2009 forest growth simulator (Diéguez-Aranda et al. 2009) to facilitate its use by forest managers.

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