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COMPARISON OF PROBABILITY DISTRIBUTION FUNCTIONS APPLIED TO TREE DIAMETER AND HEIGHT OF THREE DEVELOPMENT STAGES IN A MIXED BEECH (*FAGUS ORIENTALIS* LIPSKY) FOREST IN HYRCANEAN REGION OF IRAN

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Abstract

Identification and assessment of quantitative characteristics of forest communities are the basic components of forest planning. Modeling frequency distributions of tree parameters such as diameter and height in various fields of forest sciences, including forest management, silviculture and forest biometrics plays an important role. The objective of this study was to determine the best probability distribution functions of tree diameter and height in each of three development stages (i.e., initial, optimal, and decay) in an unmanaged, uneven-aged mixed beech (*Fagus orientalis* Lipsky) forest. For this purpose, three sample plots of one hectare (100 m × 100 m), one in each of three development stages, were selected in the Hyrcanean region of Iran. Diameter and height of all trees with a diameter at breast height larger than 7.5 cm were measured. Beta, Johnson's S_B, lognormal, gamma and Weibull probability distribution functions were fitted to each diameter and height distribution. The comparison of observed and predicted probabilities was performed using Kolmogorov-Smirnov, Anderson-Darling and Chi-square tests. The goodness of fit of each distribution function varied among development stages. The three-parameter lognormal, the three-parameter Weibull and the Johnson's S_B functions showed the highest goodness of fit for tree diameter distribution at the initial, optimal and decay stages, respectively. For tree height distribution, the Johnson's S_B was the best fitted function for the optimal and decay stages, while none of these functions adequately explained the height distribution of trees at the initial stage. Best fitted models can be used to characterize the diameter and height distributions of similar stands and help achieve an optimal structure.

Key word: diameter and height distribution, forest development stage, Iran, mixed beech stand, probability distribution functions.

Introduction

Hyrcanean forests along the north-facing aspects of Elborz Mountains in Northern Iran are from natural origin and generally form uneven-aged stands composed of hardwood deciduous species (Sagheb-Talebi et al. 2004). Five main plant communities (i.e., *Querco-Buxetum*, *Querco-Carpinetum*, *Parrotio-Carpinetum*, *Fagetum* *hyrcanum*, and *Carpinetum orientale*) are distinguished along the altitudinal gradient of Elborz Mountains (Sagheb-Talebi et al. 2004). As for natural European beech (*Fagus sylvatica* L.), Oriental beech (*Fagus orientalis* Lipsky) forests are submitted to a wind disturbance regime that leads to a fine-scaled mosaic pattern of forests (Korpel 1995). This mosaic is composed of forest patches of three recognized development stages (i.e. the decay, initial, and optimal stages) that are spatially adjacent to each other or are developing in a cycle over time (Leibundgut 1993, Korpel 1995).

At the initial stage, the high growth rate of young trees produces a sustained increase in stand volume while canopy gaps are gradually closed. Although the stand structure at the early initial stage is still uneven-aged and multilayered, it changes gradually until the optimal stage is reached and then becomes regular and even-aged. The optimal stage is also characterized by a closed canopy, a scarce regeneration stratum and the presence of numerous suppressed small trees. At the decay stage, the increasing presence of dead wood and the establishment of tree regeneration in canopy gaps create vertical irregularities and uneven-agedness. Later, stem density and stand volume continue to decrease as a result of the death of many old trees that are replaced by a large number of young trees, and the forest gradually returns to the initial stage (Korpel 1995, Emborg et al. 2000). Each development stage is thus associated with a specific stand structure which can be represented by a tree diameter distribution (Es-hagh Nimvari and Mataji 2014).

Forest management based on close-to-nature silvicultural systems requires the characterization of forest resources for an adequate planning of future silvicultural interventions. Diameter distribution is a simple and yet powerful feature for characterizing forest stands and representing their sustainability potential (Bailey and Dell 1972, Rubin et al. 2006). For example, this tool can be used to predict whether the density of smaller trees is sufficient to replace the current population of larger trees in the future stand (Rubin et al. 2006). Also, knowledge of tree diameter distribution and its dependence on site, stand composition, age, and density is useful for economic and biological purposes (Bailey and Dell 1972). On one hand, knowing the number of trees in different diameter classes is essential to manipulate the stand structure and composition close to those of primeval forests. On the other hand, tree diameter distribution can be applied to determine the industrial use of wood, harvesting costs, expected yield, or price of the different products. Tree heights can be considered as the main morphometric variables of forest trees. In the forest biometrics, some variables such as height of forest trees can be used in several cases including the determination of tree and stand volume, site index, form factor, and slenderness coefficient 76 (Mohammadalizade et al. 2013). Awareness of tree diameter and height distributions are generally required for effective forest management planning and are particularly valuable in forest mensuration and inventory (Wang et al. 2008, Tsogt et al. 2013).

Many attempts have been made to describe tree diameter distribution using mathematical models: De Liocourt (1898) suggested a reversed J-shaped distribution for uneven-aged stands, which was later formulated as a negative exponential function that was widely used in forestry (Meyer 1952, Leak 1965). This negative exponential function was applied to mixed forests in Northern Pennsylvania as well as to many other forest stands (Shunzhong et al. 2006 quoted from Meyer 1952; Schmelz and Lindsey 1965). However, since that time, other statistical distribution functions were successfully applied to various stand types (Namiranian 1990). For example, the diameter distribution of Iranian beech forests has been modeled using several functions such as the beta, Weibull, normal, exponential, lognormal and gamma (e.g. Namiranian 1990, Mattaji et al. 2000, Fallah et al. 2000, Fallah et al. 2006, Mohammadalizade et al. 2009, Amanzadeh et al. 2011, Sheykholeslami et al. 2011, Fallahchai et al. 2012, Fallahchai and Shokri 2014, Mirzaei et al. 2015a). The beta function seems to be appropriate to estimate the diameter distribution of many stand types (e.g., Namiranian 1990, Mattaji et al. 2000; Liu et al. 2002). Fallahchai and Shokri (2014) concluded that the three-parameter Dagum distribution has more ability to determine the diameter distribution of *Alnus subcordata* trees.

However, Nord-Larsen and Cao (2006) considered that Weibull distribution functions are probably the most widely applied for modelling tree size distributions because of their high flexibility, while Siipilehto (1999) and Zhang et al. (2003) found that Johnson's S_B gave slightly better results than the Weibull function.

In the case of height distribution, various probability density functions (*pdf*s), e.g., Bur, Dagum, beta, normal, gamma, exponential, Johnson's S_B and Weibull have been successfully used (Tsogt et al. 2013, Kängsepp et al. 2014, Mohammadalizade et al. 2013, Mirzaei et al. 2015a, Mirzaei et al. 2015b). Tsogt et al. (2013) studied the height distribution of even-aged Scots pine (*Pinus sylvestris* L.) forests at three different age-classes (young, juvenile and old). They concluded that for the stand with the normal distribution shape, Johnson's S_B was better than Burr and Dagum functions. Mirzaei et al. (2015b) found that the beta and Weibull distributions were appropriate to explain the distributions of tree height, a diameter at breast height (DBH) and crown area.

Mixed beech forests are the richest and the most dominant tree community on the middle lands of Hyrcanean forests of Iran and as such, have the highest commercial value. In them, no studies were realized to model diameter and height distributions of natural mixed forests with beech and hornbeam predominance representing different development stages using probability density functions (*pdf*s). The specific objective of this study was to compare different statistical functions for describing tree diameter and height distributions of three forest development stages (initial, optimal, and decay) in an untouched uneven-aged mixed beech stands in the Hyrcanean region of Iran. The best fitted distributions in each of the three studied development stages can be used to effectively apply close to nature interventions in managed stands of similar composition in order to achieve stand stability and optimum biological and economic production.

Material and Methods

Study area

The study site was located in Caspian forest of Sistan district No 4 of Zeilaki watershed No 23 in Northern Iran (Longitudes 49°49´4″ and 49°53´15″, Latitudes 36°55´10″ and 36°58′16"). This study was conducted in a 59-ha forest area with an elevation range between 350 and 750 m a.s.l., an average annual temperature of 14 °C and an average annual precipitation 1273 mm. Mean stand density and growing stock volume are 212 stems ha^{-1} and 299.6 m^3 ha^{-1}, respectively. Precipitation is well distributed so that there is generally no drought periods during a year. The soils are acidic brown or forest brown with soil textures varying from clay to clay-loam, mostly with single grain-soil structure and low permeability rate. The study area consists of a natural, mixed, deciduous forest dominated by (beech) *Fagus orientalis* and with less proportion of hornbeam (*Carpinus betulus* L.) minor presence of maple (*Acer velutinum* Boiss.), coliseum maple (*Acer cappadocicum* Gled.), caucasian alder (*Alnus subcordata* C.A.Mey), elm (*Ulmus glabra* Huds.), persimmon (*Diospyros lotus* L.), ironwood (*Parrotia persica* (Dc.) C.A. Mey) and lime (*Tilia begonifolia* Stev). Because the study area is subjected to periodic events of wind disturbances, it is composed of a mosaic of small forest patches, each corresponding to a specific development stage, but forming an uneven-aged forest when considered together. The study area has developed with minimal human disturbances and without silvicultural interventions during the last decades. Hence, this forest represents an untouched and unmanaged natural forest typical of native ancient forests of this region.

Data collection

We first conducted a forest survey in the study area to locate forest patches corresponding to different development stages. To determine them for each forest patch, the same criteria as those of previous studies were used (Korpel 1995, Sagheb-Talebi et al. 2003, Delfan Abazari et al. 2004, Mataji and Sagheb-Talebi 2007, Sefidi and Marvie Mohadjer 2010, Akhavan et al. 2012, Zenner et al. 2015). These criteria were the number and volume of stems, dead wood volume, proportion of dead wood volume by diameter class, proportion of volume by diameter class, presence of canopy gaps and number of tree layers (Table 1). Then, three forest patches were randomly selected to establish 1-ha (100 m × 100 m) sample plots during the summer of 2014 such as one plot was established in each of three develop-

Table 1. **Minimum criteria for identification of plots pursuant to development stages.**

Criterion	Development stages		
	Initial	**Optimal**	**Decay**
General age estimation	Young	Middle-aged	Old
Number of stand stories	> 2	Usually 1–2	> 2
Stem density	High	Medium	Low
Highest Proportion of trees by canopy stratum	In lower and middle stories	In upper story	In middle and upper stories
Highest proportion of trees by size	In small and medium sizes	In medium and large sizes	In large and extra-large sizes
Stand volume	Medium	High	Low
Dead wood volume	Medium	Low	High
Highest proportion of dead wood volume by size	In large and extra-large sizes	In small and medium sizes	In extra-large sizes
Gap	Present	Usually absent	Present
Regeneration	Clustered and present in gaps	Little and scattered over the whole area	Clustered and present in gaps

Note: Stage of development of plots was identified according to many of these criteria as possible.

ment stages (initial, optimal, and decay). Previous studies have confirmed the appropriateness of 1-ha plot to study the structure of Oriental beech stands in Iran (Eslami and Sagheb-Talebi 2007, Sagheb-Talebi and Schùtz 2002). Plots were all established in mixed beech stands with the same local climate, elevation, soils, aspect and slope. These plots had eastern aspect with an average slope of about of 45 %. In each plot, the species, height and diameter of all trees with DBH larger than 7.5 cm were recorded.

Data analysis

Five probability distribution functions (*pdf*s) were tested for characterizing the empirical DBH and height distribution of each forest patch. These *pdf*s were the beta, Johnson's S_B, three-parameter gamma, three-parameter lognormal and three-parameter Weibull (Table 2). These theoretical distributions were selected because:

1) They were often used in other studies to explain empirical diameter and height distributions (Amanzadeh et al. 2011, Amanzadeh 2015, Mohammadalizadeh et al. 2009, Podlaski 2006, Nord-Larsen and Cao 2006, Bullock and Boone 2007, Mohammadalizade et al. 2013, Mirzaei et al. 2015a);

(2) They generally provided accurate estimations of empirical diameter distribution in mixed forests of various structures (Amanzadeh 2015, Mattaji et al. 2000, Podlaski 2006, Shunzhong et al. 2006, Mohammadalizade et al. 2013, Mirzaei et al. 2015a); and

(3) The computation of their parameters is relatively easy (Kamziah et al. 1999, Podlaski 2006).

Parameters of *pdf*s were estimated with the maximum-likelihood method

Table 2. Probability distribution functions applied in the study area.

Distribution	Density function	Parameter
Beta	$f(x)=\dfrac{1}{\beta(\alpha_1,\alpha_2)}\dfrac{(x-a)^{\alpha_1-1}(b-x)^{\alpha_2-1}}{(b-a)^{\alpha_1+\alpha_2-1}}$ $a\le x\le b,\ \alpha_1>0,\ \alpha_2>0,$	α_1 and α_2 : continuous shape parameters a and b: continuous boundary parameters β is the beta function
Johnson's S_B	$f(x)=\dfrac{\delta}{\lambda\sqrt{2\pi}z(1-z)}\exp\left(-\dfrac{1}{2}\left(\gamma+\delta\ln\left(\dfrac{z}{1-z}\right)\right)^2\right)$ $z=\dfrac{x-\xi}{\lambda},\ \xi\le x\le\xi+\lambda,\ \delta>0,\ \lambda>0$	ξ : continuous location parameter λ : continuous scale parameter δ and γ : continuous shape parameters
Gamma	$f(x)=\dfrac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}\exp(-(x-\gamma)/\beta)$ $\alpha>0,\ \beta>0,\ \gamma\le x<+\infty$	α : continuous shape parameters β : continuous scale parameter γ : continuous location parameter Γ is the gamma function
Lognormal	$f(x)=\dfrac{\exp\left(-\dfrac{1}{2}\left(\dfrac{\ln(x-\gamma)-\mu}{\sigma}\right)^2\right)}{(x-\gamma)\sigma\sqrt{2\pi}}$ $\sigma>0,\ \gamma<x<+\infty$	μ and σ : continuous parameters γ : continuous location parameter
Weibull	$f(x)=\dfrac{\alpha}{\beta}\left(\dfrac{x-\gamma}{\beta}\right)^{\alpha-1}\exp\left(-\left(\dfrac{x-\gamma}{\beta}\right)^{\alpha}\right)$ $\alpha>0,\ \beta>0,\ \gamma\le x<+\infty$	α : continuous shape parameters β : continuous scale parameter γ : continuous location parameter

by using the distribution fitting software Easyfit 5.6 Professional (MathWave Technologies 2015) that was already used in several studies (e.g. Amanzadeh et al. 2011, Sohrabi and Taheri 2012, Khongor et al. 2011, Tsogt et al. 2013, Humphrey and Godwin 2014, Mirzaei et al. 2015a).

Comparison of actual DBH distributions with the theoretical *pdf* were evaluated by statistical tests that included Kolomogorov-Smirnov (K-S), Anderson-Darling (A-D), and χ^2 goodness-of-fit tests (Nord-Larsen and Cao 2006, Khongor et al. 2011, Tsogt et al. 2013, Amanzadeh et al. 2011, Humphrey and Godwin 2014).

The null and the alternative hypotheses in these tests were:

- H_0 – the data follow the specified distribution;
- H_1 – the data do not follow the specified distribution.

Kolmogorov-Smirnov statistic (Chakravarti et al. 1967) is a nonparametric test that is applicable when the population distribution function is continuous and can thus be used to investigate the significance of the difference between an observed distribution and the theoretical probability distribution (Kanji 2006, Jäntschi and Bolboaca 2009). It is based on the maximum vertical difference between the theoretical and the empirical cumulative distribution function (ECDF). The K-S statistic (D) corresponds to:

$$D = \max (F - Sn),$$

where F is the observed cumulative frequencies and Sn is the theoretical cumulative frequencies.

A-D test (Stephens 1974) is also a nonparametric test that is used to test if a data sample came from a population with a specific distribution. It is a modification of K-S and gives more weight to the tails of a distribution than does K-S test. A-D statistic (A^2) is defined as:

$$A^2 = -N - S,$$

where N is the sample size,

$$S = \sum_{i=1}^{N} \frac{(2i-1)}{N}\left[\ln F(X_i) + \ln\left(1 - F\left(X_{N+1-i}\right)\right)\right]$$

and F is the cumulative distribution function of the specified distribution. Note that X_i are the ordered data.

The χ^2 test (Snedecor and Cochran 1989) is used to test if a data sample coming from a population with a specific distribution. This test is applied to categorical data (i.e. data put into classes). This is actually not a restriction since classes can be easily formed from continuous data before applying the chi-square test. However, the value of the χ^2 test statistic is dependent on how the data are classified. Another disadvantage of the chi-square test is that it requires a sufficient sample size in order for the chi-square approximation to be valid. This test is sensitive to the choice of classes. For the χ^2 approximation to be valid, the expected frequency in each class should be at least five or more data points. This test is not valid for small samples, and if some of the counts are less than five, classes must be combined.

The χ^2 test statistic is defined as:

$$x^2 = \sum_{i=1}^{k} \left(O_i - E_i\right)^2 \Big/ E_i$$

where O_i is the observed frequency for class i and E_i is the expected frequency for class i.

Hypothesis tests of the distribution functions were performed by examining the *p* value that was associated with a goodness-of-fit statistic. If the test statistic was greater than the critical value obtained from a table or the *p* value was less than a predefined critical value, the null hypothesis (H_0) was rejected at the chosen significance level (0.01, 0.05, etc.), and accepted functions were ranked according to the test statistic.

The stem density of *F. orientalis* was higher than that of *C. betulus* in all development stages while the DBH of *F. orientalis* was larger than that of *C. betulus* except in the initial stage, indicating that small diameter trees in this stage were mostly *F. orientalis* (Table 3). The greatest coefficient of variation of DBH occurred in the initial stage, while in the op-

Results

In total, the diameter and height of 857 trees in the three sampled plots were measured. The initial stage had the highest frequency in small trees whereas the decay stage was mostly composed of mature trees. As expected, the optimal stage was associated with a bell-shaped, skewed distribution (Fig. 1 and 2). Mean tree DBH and height increased from initial to decay stages, while the number of trees followed the opposite trend (Table 3). The initial stage was associated with the largest stem number, and the smallest average DBH. The decay stage can be recognized by the lowest stem number and basal area (Tables 3 and 4).

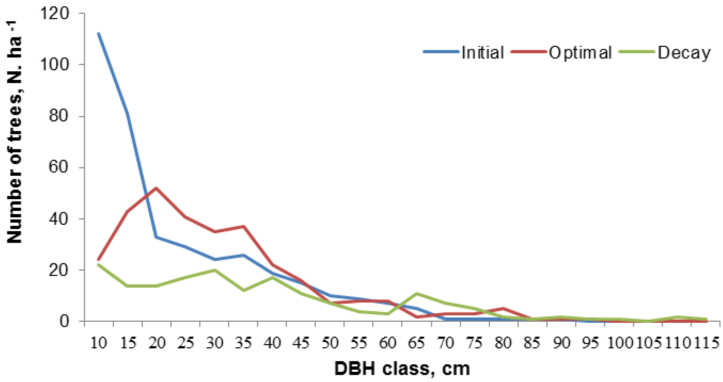

Fig. 1. Stem diameter distribution in each developmental stage.

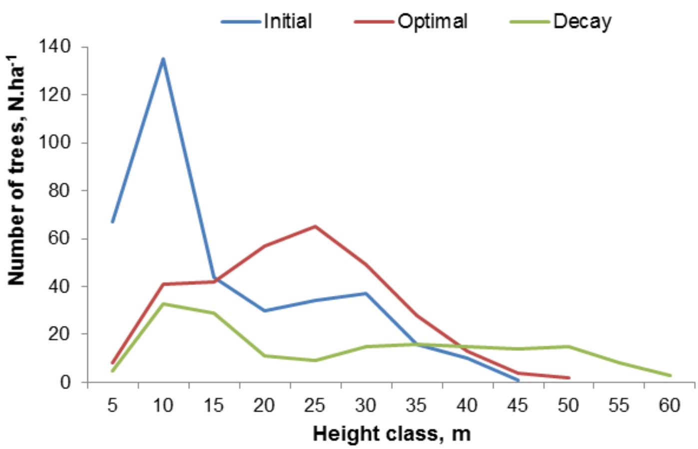

Fig. 2. Stem height distribution in each developmental stage.

Table 3. Species characteristics of each development stage (based on 1 ha area).

Indicator	Sample plot											
	Initial				Optimal				Decay			
	Beech	Hornbeam	Others	Total	Beech	Hornbeam	Others	Total	Beech	Hornbeam	Others	Total
Number of stems	217	127	30	374	175	132	2	309	91	57	26	174
Frequency, %	58	34	8	100	56.6	42.7	0.7	100	52.3	32.7	14.9	100
Mean dbh, cm	20	28	34	23.8	31	27.5	30	30.4	40	36	36	38
DBH CV, %	68	51	71	66	57	45	-	54	72	41	48	61
Mean height, m	14	18	20.5	15.9	23.7	22.1	23	23	27	27.5	31	27.8
Basal area, $m^2 \cdot ha^{-1}$	15.04	14.61	4.25	33.9	19.3	9.4	0.18	28.9	17	7	3.25	27.25
Volume, $m^3 \cdot ha^{-1}$	160.6	107.7	46.7	315	246.9	86.8	1.7	335.4	211.5	64.5	29.6	305.6
Number of snags	2	1	1	4	1	6	-	7	4	5	-	9
Number of snags60 cm dbh	-	-	-	-	1	1	-	2	2	-	-	2

Note: Beech (*Fagus orientalis*); Hornbeam (*Carpinus betulus*) and others, tree species including *Acer velutinum*, *Acer cappadocicum*, *Alnus subcordata*, *Ulmus glabra*, *Diospyros lotus*, *Parrotia persica* and *Tilia begonifolia*; CV coefficient of variation.

timal stage, all trees had about the same height and DBH, as indicated by the lowest CV of DBH and height (Table 4). In the decay stage due to the establishment of regeneration in gaps created by dead standing or downed mature trees, CV of DBH and height were quite large (Table 4). In addition, the initial stage was characterized by the smallest DBH standard deviation (Table 4). The greatest diameter and height variation occurred in the decay stage (Table 4). Tree distributions displayed positive skewness, i.e., asymmetry towards positive values for both DBH and height, positive kurtosis for DBH and negative kurtosis for height at all three stages (Table 4). The greatest asymmetry occurred in diameter and height distributions at the initial stage, while the optimal stage was associated with the less asymmetrical height distribution (Table 4).

The results of the goodness-of-fit tests for each of the five distribution functions applied to the observed DBH and height data of the initial stage are presented in Table 5. K-S and A-D tests indicate that the three-parameter lognormal function can adequately fit the observed DBH as suggested by their lower calculated statistics compared to the tabulated critical values. Other distribution functions were unable to adequately represent tree diameter distribution at this stage. Using DBH classes of 5 cm, Figure 3 shows the observed DBH distribution and the estimated probabilities of the distribution functions of DBH class. None of

Table 4. Descriptive statistics for DBH and height of each development stage (based on 1 ha area).

Development stages	Sample size	Variable	Mean	Standard deviation	Maximum	Minimum	Coefficient of variation, %	Skewness	Kurtosis
Initial	374	DBH, cm	23.8	15.71	91	5	66	1.36	1.55
		Height, m	15.9	9.77	43	3	61.6	0.92	-0.32
Optimal	309	DBH, cm	30.4	16.4	95	5	54	1.31	1.86
		Height, m	23	9.24	52	3	40	0.24	-0.28
Decay	174	DBH, cm	38	23.3	115	8	61	1.00	0.67
		Height, m	27.8	15.5	60	5	56	0.31	-1.2

the distribution functions were able to adequately fit the height observations of the stand representing the initial stage (Table 5). Using height classes of 5 m, Figure 4 shows the observed height distribution and the estimated probabilities of the five distribution functions.

Table 6 presents the parameter values of the distribution models illustrated in Figures 3 and 4.

Table 7 presents the results of goodness-of-fit tests of each DBH and height distribution function for the optimal stage. All three tests indicated that all distributions were significantly related to the observed DBH data. Among these five functions, K-S and A-D tests indicate that Johnson's S_B and three-parameter gamma distributions were more closely related to the observed data as indicated by the lower values of their statistics. However, the χ^2 test indicated that the three-parameter Weibull distribution

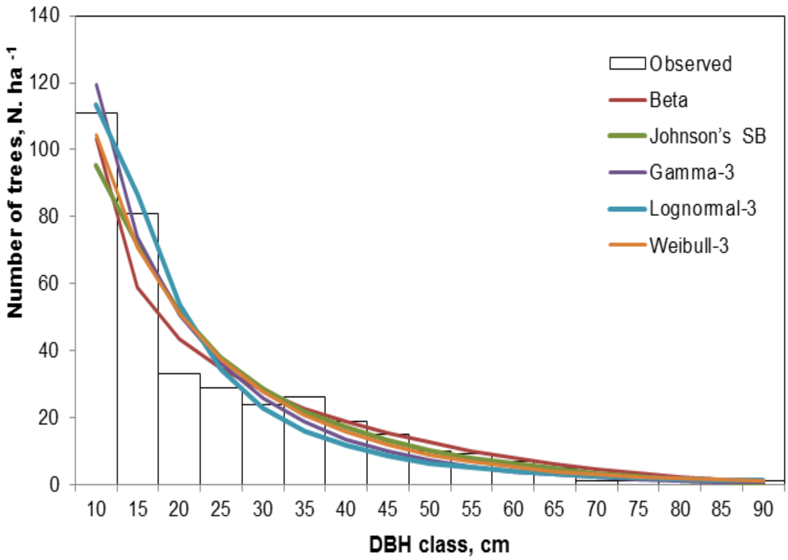

Fig. 3. Actual and theoretical DBH distributions in the initial stage.

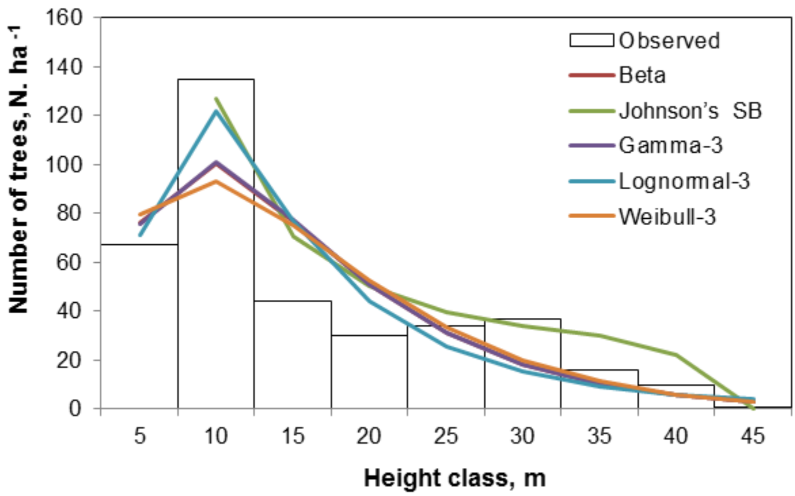

Fig. 4. Actual and theoretical height distributions in the initial stage.

Table 5. Goodness-of-fit and ranking of the five distributions in the initial stage.

Variable	Distribution	Kolmogorov Smirnov			Anderson Darling			Chi-squared		
		Statistic	*P* value	Rank	Statistic	Critical value	Rank	Statistic	*P* value	Rank
DBH	Beta	0.12431	0.00**		10.962	3.9074**		60.556	0.00**	
	Johnson's S_B	0.10603	0.00**		3.742	3.9074 *ns*	2	16.374	0.03 *ns*	1
	Gamma-3	0.11031	0.00**		46.607	3.9074**		-	-	
	Lognormal-3	0.08413	0.055 *ns*	1	3.1582	3.9074 *ns*	1	58.302	0.00**	
	Weibull-3	0.09542	0.00**		44.793	3.9074**		-	-	
Height	Beta	0.12047	0.00**		6.8092	3.9074**		60.323	0.00**	
	Johnson's S_B	0.09666	0.00**		58.681	3.9074**		-	-	
	Gamma-3	0.1207	0.00**		6.8988	3.9074**		71.269	0.00**	
	Lognormal-3	0.09513	0.00**		4.9316	3.9074**		55.134	0.00**	
	Weibull-3	0.1245	0.00**		7.2893	3.9074**		112.04	0.00**	

Note: * & ** – the null hypothesis rejected at α = 0.05 and α = 0.01, respectively and ns – no significant.

Table 6. Distribution parameter estimates for initial stage.

Variable	Distribution	Parameters			
DBH	Beta	$\alpha_1 = 0.63035$	$\alpha_2 = 2.6744$	$a = 8.0$	$b = 100.1$
	Johnson's S_B	$\gamma = 1.4399$	$\delta = 0.79781$	$\lambda = 90.967$	$\xi = 5.9557$
	Gamma-3	$\alpha = 0.85437$	$\beta = 16.961$	$\gamma = 8.0$	
	Lognormal-3	$\sigma = 1.0367$	$\mu = 2.3402$	$\gamma = 7.1483$	
	Weibull-3	$\alpha = 0.93895$	$\beta = 16.404$	$\gamma = 8.0$	
Height	Beta	$\alpha_1 = 1.7889$	$\alpha_2 = 2.3206E+6$	$a = 2.928$	$b = 1.6822E+6$
	Johnson's S_B	$\gamma = 0.75061$	$\delta = 0.50582$	$\lambda = 35.687$	$\xi = 5.669$
	Gamma-3	$\alpha = 1.8029$	$\beta = 7.1779$	$\gamma = 2.9255$	
	Lognormal-3	$\sigma = 0.75145$	$\mu = 2.3289$	$\gamma = 2.4261$	
	Weibull-3	$\alpha = 1.3736$	$\beta = 14.174$	$\gamma = 2.9718$	

was more closely related to the observed data. Figure 5 shows the observed data and the estimated probability functions of DBH class whose parameter values are presented in Table 8. K-S and A-D tests indicate that Johnson's S_B and three-parameter Weibull function can adequately fit the observed height data as suggested by their *P* value and by lower calculated statistics compared to tabulated critical values (Table 7). However, the χ^2 test indicated that the three-parameter Gamma and lognormal distribution were not able to fit height observation (Table 7). Figure 6 shows the observed data and the estimated probability functions of height class whose parameter values are presented in Table 8.

The results of goodness-of-fit tests of the distribution functions applied to the decay stage are shown in Table 9. K-S and A-D tests indicated that all distribution functions can significantly fit the actual di-

Table 7. Goodness-of-fit and ranking of the five distributions in the optimal stage.

Variable	Distribution	Kolmogorov Smirnov			Anderson Darling			Chi-squared		
		Statistic	*P* value	Rank	Statistic	Critical value	Rank	Statistic	*P* value	Rank
DBH	Beta	0.04936	0.42 *ns*	4	0.40794	3.9074 *ns*	2	11.306	0.18 *ns*	3
	Johnson's S_B	0.04362	0.58 *ns*	1	0.77261	3.9074 *ns*	5	9.3294	0.31 *ns*	2
	Gamma-3	0.04934	0.42 *ns*	3	0.39815	3.9074 *ns*	1	11.321	0.18 *ns*	4
	Lognormal-3	0.05415	0.31 *ns*	5	0.51561	3.9074 *ns*	3	11.789	0.16 *ns*	5
	Weibull-3	0.04769	0.46 *ns*	2	0.66136	3.9074 *ns*	4	8.027	0.43 *ns*	1
Height	Beta	0.05644	0.27 *ns*	3	0.72219	3.9074 *ns*	3	13.24	0.10 *ns*	3
	Johnson's S_B	0.05032	0.40 *ns*	1	0.65684	3.9074 *ns*	1	12.187	0.14 *ns*	1
	Gamma-3	0.06043	0.20 *ns*	5	0.94972	3.9074 *ns*	5	17.537	0.02*	
	Lognormal-3	0.05825	0.23 *ns*	4	0.91235	3.9074 *ns*	4	17.259	0.02*	
	Weibull-3	0.05397	0.05 *ns*	2	0.68596	3.9074 *ns*	2	12.501	0.13 *ns*	2

Note: * & ** – the null hypothesis rejected at α = 0.05 and α = 0.01, respectively and ns – no significant.

ameter data. However, for all three goodness-of-fit tests, Johnson's S_B distribution was more closely related to the observed DBH distribution. Figure 7 shows the ob-

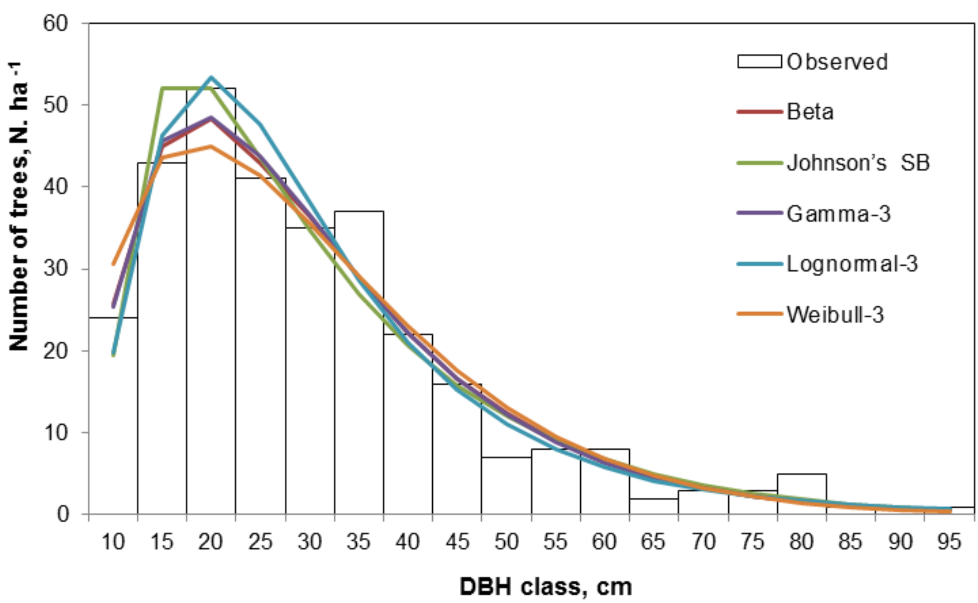

Fig. 5. Actual and theoretical DBH distributions in the optimal stage.

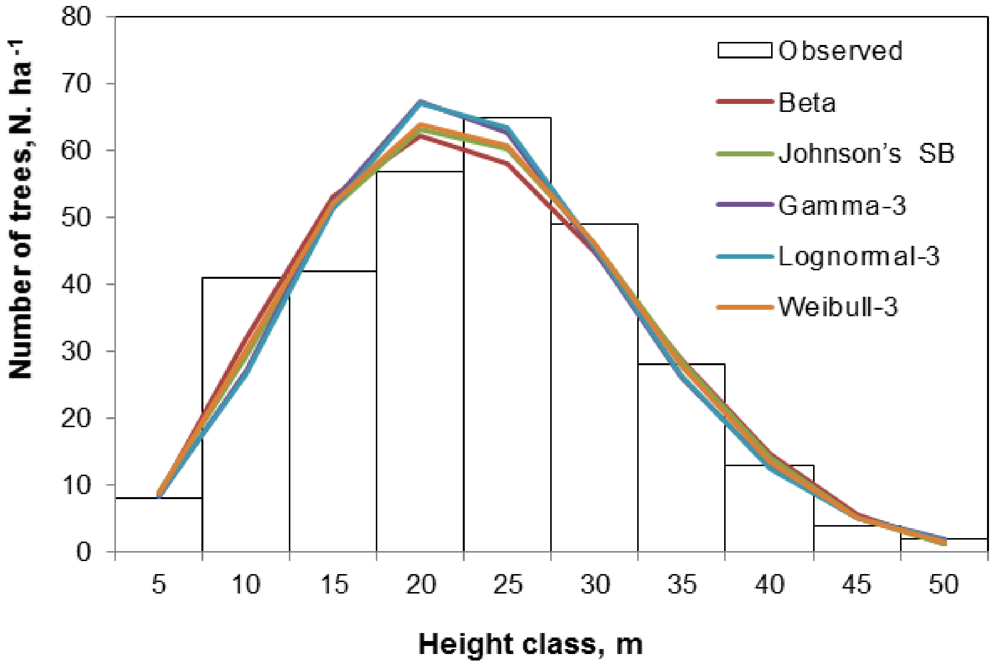

Fig. 6. Actual and theoretical height distributions in the optimal stage.

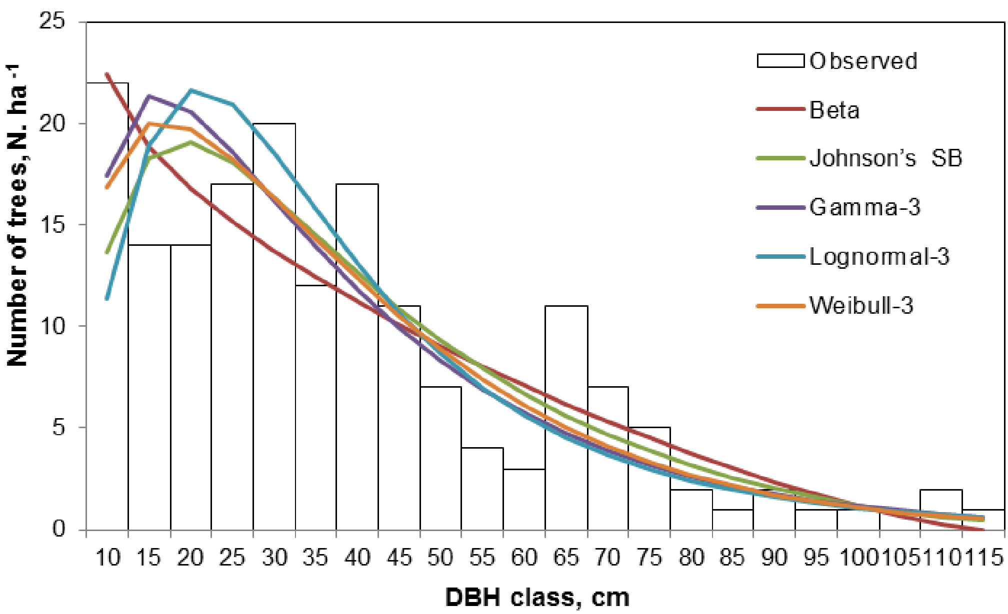

Fig. 7. Actual and theoretical DBH distributions in the decay stage.

Table 8. Distribution parameter estimates for optimal stage.

Variable	Distribution	Parameters			
DBH	Beta	$\alpha_1 = 1.9447$	$\alpha_2 = 119.94$	$a = 7.2644$	$b = 1456.6$
	Johnson's S_B	$\gamma = 1.9088$	$\delta = 1.0811$	$\lambda = 128.14$	$\xi = 7.3804$
	Gamma-3	$\alpha = 1.9867$	$\beta = 11.664$	$\gamma = 7.2145$	
	Lognormal-3	$\sigma = 0.57173$	$\mu = 3.1762$	$\gamma = 2.2901$	
	Weibull-3	$\alpha = 1.4225$	$\beta = 24.92$	$\gamma = 7.7553$	
Height	Beta	$\alpha_1 = 3.2793$	$\alpha_2 = 5.5323$	$a = 0.66128$	$b = 60.746$
	Johnson's S_B	$\gamma = 0.80996$	$\delta = 1.9361$	$\lambda = 78.677$	$\xi = 8.6517$
	Gamma-3	$\alpha = 27.271$	$\beta = 7.1779$	$\gamma = -25.373$	
	Lognormal-3	$\sigma = 0.11346$	$\mu = 4.3916$	$\gamma = -58.253$	
	Weibull-3	$\alpha = 2.5889$	$\beta = 24.61$	$\gamma = 1.1972$	

Table 9. Goodness-of-fit and ranking of the five distributions in the decay stage.

Variable	Distribution	Kolmogorov Smirnov			Anderson Darling			Chi-squared		
		Statistic	*P* value	Rank	Statistic	Critical value	Rank	Statistic	*P* value	Rank
DBH	Beta	0.0844	0.16 ^{ns}	4	1.138	3.9074 ^{ns}	3	7.5172	0.37 ^{ns}	2
	Johnson's S_B	0.06807	0.37 ^{ns}	1	0.67241	3.9074 ^{ns}	1	5.6985	0.57 ^{ns}	1
	Gamma-3	0.09474	0.08 ^{ns}	5	1.1721	3.9074 ^{ns}	4	22.42	0.00 **	
	Lognormal-3	0.07253	0.30 ^{ns}	2	1.233	3.9074 ^{ns}	5	10.213	0.17 ^{ns}	3
	Weibull-3	0.07944	0.21 ^{ns}	3	0.93211	3.9074 ^{ns}	2	14.973	0.03 *	
Height	Beta	0.081113	0.19 ^{ns}	2	9.1416	3.9074 **		-	-	
	Johnson's S_B	0.05938	0.55 ^{ns}	1	12.241	3.9074 **		-	-	
	Gamma-3	0.11135	0.02 *		3.0931	3.9074 *		24.974	0.00 **	
	Lognormal-3	0.11939	0.01 *		3.8496	3.9074 *		38.645	0.00 **	
	Weibull-3	0.02034	0.02 *		3.1169	3.9074 *		37.434	0.00 **	

Note: * & ** – the null hypothesis rejected at α = 0.05 and α = 0.01, respectively and ns – no significant.

served and estimated values for the dist ribution functions whose parameters are presented in Table 10. A-D and χ^2 tests indicate that none of the distribution functions were able to fit height observations, while K-S test indicated that the Johnson's S_B and beta distributions were significantly related to the observed data (Table 9). Figure 8 shows the observed data and the estimated probability functions of height class whose parameter values are presented in Table 10.

Discussion

Stand development stages are generally characterized by several structural

Table 10. Distribution parameter estimates for decay stage.

Variable	Distribution	Parameters			
DBH	Beta	$\alpha_1 = 0.9149$	$\alpha_2 = 2.3462$	$a = 8.0$	$b = 115.0$
	Johnson's S_B	$\gamma = 1.422$	$\delta = 1.0443$	$\lambda = 148.29$	$\xi = 2.4148$
	Gamma-3	$\alpha = 1.3551$	$\beta = 22.259$	$\gamma = 7.837$	
	Lognormal-3	$\sigma = 0.62824$	$\mu = 3.4664$	$\gamma = -0.55885$	
	Weibull-3	$\alpha = 1.2399$	$\beta = 32.169$	$\gamma = 7.895$	
Height	Beta	$\alpha_1 = 0.85416$	$\alpha_2 = 1.2105$	$a = 5.0$	$b = 60.041$
	Johnson's S_B	$\gamma = 0.26519$	$\delta = 0.48972$	$\lambda = 49.791$	$\xi = 6.8766$
	Gamma-3	$\alpha = 1.7581$	$\beta = 13.283$	$\gamma = 4.4161$	
	Lognormal-3	$\sigma = 0.58752$	$\mu = 3.2198$	$\gamma = -1.5762$	
	Weibull-3	$\alpha = 1.4483$	$\beta = 25.451$	$\gamma = 4.6297$	

and biological criteria such as species composition and distribution, horizontal and vertical structure, canopy status, forest health and rate of tree mortality (Chernyavskyy 2003, Hassani and Amani 2010). Distribution functions are powerful mathematical tools for describing forest stand diameter structure (Mohammadalizadeh et al. 2009, Duan et al. 2013) and predicting the growth of stands of various structures (even- and uneven-aged) and compositions (pure and mixed) (Palahi et al. 2007, Gorgoso et al. 2007). This study was carried out to compare the performance of different probability distribution functions in estimating the diameter and height distribution of trees from different development stages in uneven-aged natural mixed beech forest. Fitting results showed that the best theoretical distributions differed between development stages. Structural differences between development stages can explain these different results.

At the initial stage, the observed DBH distribution was characterized by few trees larger than 70 cm in DBH, a high proportion of trees with DBH below 20 cm and the greatest coefficient of variation of DBH (Fig. 1, Table 4). The wide range of DBH and the reverse-J shape of the actual DBH distribution indicate that the initial stage was characterized by an irregular uneven-aged structure. For such a diameter distribution, three-parameter lognormal and Johnson's S_B distribution were the best fitted functions. These functions resulted in positively skewed distributions and their parametric distributions were very flexible (Hafley and Schreuder 1977, Zwillinger and Kokoska 2000). Both functions were thus able to take into account the presence of many small trees together with some large trees that displayed asymmetry towards positive values (Table 4). Similar results were obtained by Amanzadeh et al. (2011) in natural beech stands and by Amanzadeh (2015) in natural mixed hornbeam forests of Iran, both at the initial development stage. Nanang (1998) also observed that lognormal distribution was appropriate to represent the diameter distribution of Neem (*Azadirachta indica* A. Juss. [Meliaceae]) using the K-S test. In addition, Sheykholeslami et al. (2011) observed the good performance of the lognormal function based on the results of K-S and χ^2 test in uneven-aged forests of Iran, but without taking into account different development stages. The tree height distribution at the initial stage was characterized by an outstanding peak in the 10-m height class and a marked positive skewness (Fig. 4). All modeled functions displayed a peak around the 10-m height class but they overestimated the tree frequency observed in height classes 15 and 20 and underestimated that observed in classes 25, 30, 35 and 40 m, while the Johnson's S_B followed the opposite trend (Fig. 4). In this stage, none of the tested functions adequately fit the height distribution likely because the competition dynamics among trees for light stimulated the high growth of young trees (Akhavan et al. 2012).

The empirical DBH distribution at the optimal stage was characterized by a large number of trees with DBH below 35 cm and by a moderate proportion of medium diameter classes (Fig. 1 and 5). Overall, homogeneous diameter and height distributions (lowest DBH and height CV, Table 4) were observed at this stage which was very similar to even-aged stand. Tree DBH distribution displayed a peak at the 20 cm class with a positive skewness. The DBH distribution was best fitted by the three-parameter Weibull, Johnson's S_B and three-parameter gamma functions and, to a lesser degree, by the beta and three-parameter lognormal functions. This is in agreement with the findings of Amanzadeh et al. (2011) who obtained similar appropriate models for fitting the diameter distribution at this stage although the ranking of these functions was different. Similarly, Amanzadeh (2015) reported that Weibull, gamma, and lognormal distribution functions were more closely related to the diameter distribution of that type of stand. Consequently, Weibull and gamma functions seem to have the greatest flexibility and universality to represent the diameter distribution of trees at the optimal stage. Tree height distribution displayed a peak at the 25 m class, whereas, the lower negative kurtosis of height distribution indicated that the stand representing this stage was a little bit flatter than a normal distribution (Fig. 6, Table 4). For such a height distribution, Johnson's S_B, three-parameter Weibull and beta distribution were the best fitted functions. Mohammadalizadeh et al. (2013) found that the Weibull function was more closely related to height distribution in uneven-aged stands. Mirzaei et al. (2015a) observed the good performance of the beta and Weibull distributions based on the results of K-S test in open oak forests of Iran.

At the decay stage, a higher proportion of old trees with DBH above 50 cm was observed compared to previous stages (Fig. 1 and 7). The natural mortality of some of these old trees created canopy gaps that were gradually filled by small diameter trees. Accordingly, the observed DBH distribution has a moderate downtrend indicating an uneven-aged structure. This diameter distribution was best modeled by the Johnson's S_B and, to a lesser degree, by the three-parameter lognormal, beta and three-parameter Weibull distribution. Similar results have been reported by Amanzadeh et al. (2011) and Amanzadeh (2015) who noticed the greatest usefulness of the Johnson's S_B distribution at this stage. In addition, Tesgot et al. (2013) observed that the Johnson's S_B function adequately represented the diameter distribution of old forest stands while the three-parameter Weibull function was useful in old-growth Norway spruce stands (Karczmarski 2005). The tree height distribution was characterized by an outstanding peak in classes 10–15 and by lower peaks in classes 30, 35, 40, 45 and 50 (Fig. 8). This particular height distribution observed at the decay stage was adequately fit by the Johnson's S_B and beta distributions. Mirzaei et al. (2015b) also observed that the beta distribution was appropriate to represent the height distribution of uneven-aged stand from the Ilam region of Iran.

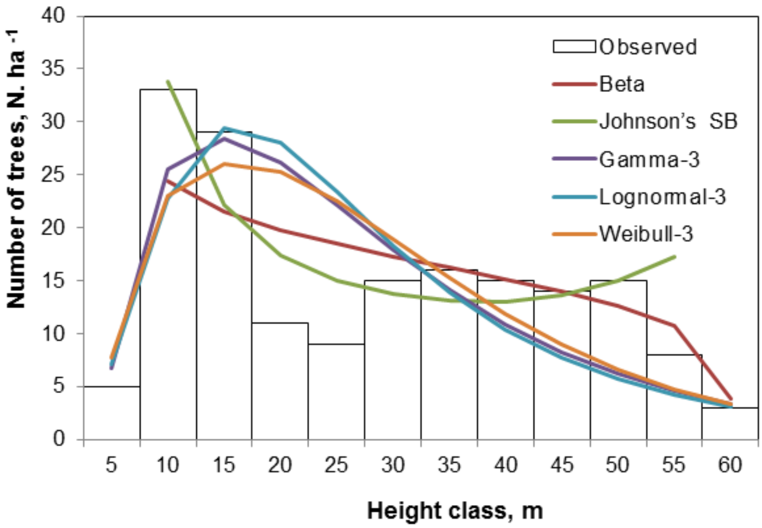

Fig. 8. Actual and theoretical height distributions in the decay stage.

Our results indicate that the three-parameter lognormal function can be appropriate for DBH distribution at all three stages in these mixed stands. Similar results have been reported by Amanzadeh et al. (2011) for the same three stages in beech stands. However, the gamma (Mohammadalizadeh et al. 2009) and the beta (Namiranian 1990, Mattaji et al. 2000, Fallah et al. 2006) distribution models were also successfully used for beech stands, but without taking into account different development stages. In this study, Beta and Johnson's S_B functions performed better to describe the height distributions of the optimal and decay stages. The Weibull distribution was found as the most appropriate model for beech stands (Mohammadalizadeh et al. 2013) while the beta distribution was the best fitted function for Persian oak (*Quercus persica* Jaub. et Spach) stands (Mirzaei et al. 2015a, Mirzaei et al. 2015b), but without taking into account different development stages.

Conclusion

Diameter distribution gives information about stand and age structure, stand stability, etc., and enables optimal planning of silvicultural treatments. In addition, the height distribution should be considered and determined simultaneously for a better interpretation of stand structure and a more accurate estimation of forest volume stocks. This information is essential for understanding how the structure of natural forests change over development stages as influenced by various factors (e.g., stem density, canopy gaps, light competition, etc.).

In accordance with our results, we conclude that a single theoretical distribution function can produce satisfactory results to model DBH distributions of stands at the optimal and decay stages. However, in stands with a large proportion of trees with DBH ≤ 20 cm (60 %) and a small proportion of trees between 20 and 30 cm in DBH (14 %) and a negative binomial distribution (e.g. initial stage), the use of a finite mixture distribution may provide accurate approximation of diameter distributions as was observed in this study. This is also the case for height distributions characterized by a large proportion of trees with height ≤ 15 m (63 %) and a small proportion of trees with height ≥ 35 (8 %). It thus seems that self-thinning is the principal process among trees in the lower and middle stories because of dense regeneration under canopy gaps at the initial stage. Consequently, the three-parameter lognormal or the finite mixture distributions can be used to effectively apply close to nature interventions (crop tree thinning) in the lower and middle stories to accelerate this natural process and, alternatively, to promote tree diversity in managed stands of similar composition. More homogenous conditions (DBH and height distributions) and high stand volume (Table 2) were observed in the optimal stage. In addition, tree regeneration density decreased because of canopy closure and sapling density declined as competition-induced mortality thinned suppressed trees. Accordingly, for both DBH and height distributions, the three-parameter Weibull and Johnson's S_B were the best model to represent this self-thinning process. However, to represent the increasing number and size of persistent canopy gaps that allow the regeneration and recruitment of shade-tolerant species and, to a lesser degree, mid-shade tolerant tree species at the decay stage, the Johnson's S_B could be used successfully for both DBH and height distributions.

Since the number, basal area and volume of *F. orientalis* trees were higher than *C. betulus* and other species at all three stages (Table 3), this dominance should be maintained if silvicultural treatments are applied (Fallah et al. 2000). Because uneven-aged mixed forests are generally resistant against natural disturbances while providing sustainable production (Alijani et al. 2014), diameter distribution functions have been widely applied to develop forest growth models. These models provide foresights of forest stand development that may simplify management decisions (Merganic and Sterba 2006).

Finally, there is no theoretical reason for the fact that a particular distribution model should be used for all situations (Wang and Rennolls 2005, Amanzadeh et al. 2011). It seems that the performance of theoretical distributions is affected by the structure (even- and uneven-aged, and pure and mixed stands), density, development stages and habitat conditions of studied stands (Bailey and Dell 1972, Amanzadeh et al. 2011, Amanzadeh 2015, Podalski 2006).

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