

VOLUME TABLES FOR THE ALEPPO PINE OF THE FOREST TATOI OF PARNITHA ATTICA (GREECE)

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Abstract

For the Aleppo pine of Tatoi forest (Parnitha, Attica, Greece), regression models that estimate the volume using diameter at breast height and/or total height were fitted. A random sample of 100 trees was taken. Eighty of these trees were used for fitting and 20 for validation. The selected model is $V = 0.267 \cdot D^2 \cdot H$ with $R^2 = 0.93$ and standard error of estimate 0.199. Comparison with a pre-existing model of 2004, revealed the similarity of the old model for the estimation of the volume of *Pinus halepensis* for the specific area.

Key words: forest management, *Pinus halepensis*, tree volume estimation.

Introduction

Pinus halepensis Mill. has a significant function in the ecology and landscape of the Mediterranean basin. This pioneer and lenient species is easily regenerated and able to colonize very low quality and degraded lands. It is an essential factor in reforestation schemes for limy soils in the arid or semi-arid climates of the Mediterranean basin, favouring the growth of broadleaf Mediterranean species (*Quercus ilex* L., *Q. coccifera* L.) within its stands (Quezel 1986). Commonly, it forms mixed stands consisting of an upperstorey of *P. halepensis* and an understorey of *Q. ilex* or *Q. coccifera* (Senni 1931, Pavari 1954, Gandullo 1972, Plaisance 1978).

The forest of Tatoi is situated on the slopes of Mount Parnitha (Attica, Greece). This area is characterized by fir forests at higher altitudes and, to a

smaller extent, by Aleppo pine and evergreen broadleaves, which spread throughout the area. In the forest there are also natural grassland of extreme importance to wildlife and agricultural crops. This area was part of the former royal estate “Tatoi”, which is now public land. The undulating terrain combined with the rich and varied vegetation, as well as the springs and the palace buildings, provide a landscape of exceptional aesthetic value.

The forest has occasionally been exploited as a part of the royal estate, so the human presence in Tatoi until 1973 was very intense (Management Plan of Parnitha National Park 1997). Nowadays, forest management has declined greatly. For both scientific and economic purposes, it is necessary to measure the existing woody material of the forest. The determination of the quantity of the produced

wood is directly related to the volume estimation of logs. Accurate volume measurement methods are not widely used due to the lengthy process and financial costs; however, sampling methods and statistical analyses are commonly used to estimate the volume of trees (Van Laar and Akça 2007).

The purpose of this research was to develop regression models to estimate the volume of Aleppo pine trees, for the Tatoi forest, with independent variables the breast height diameter and the total height, which will be compared with the model (1):

$$V_{ob} = 0.01969779 + 1.195396 \cdot V_{ib}^7 \quad (1),$$

where V_{ob} is the volume outside bark (m^3) and V_{ib} the volume inside bark (m^3), with $V_{ib} = 3.3041044 \cdot 10^{-5} \cdot D^{1.790332} \cdot H^{1.181907}$. D is the breast height diameter (cm) and H the total tree height (m). This model was taken from the Logging table of the forest, developed for the year 2004, from the Forest Service of Parnitha.

Materials and Methods

Study area

The forest of Tatoi is located 15 km north of Athens. The altitude range is 240–1033 m, with the largest part of the area having elevations ranging from 400–600 m. Land uses are given in Table 1 (Management Plan of Parnitha National Park 1997).

The forest of Tatoi has a long history starting from ancient Dhekelia, where the last phase of the Peloponnesian war was fought. Then, during the Ottoman period, it was a hideout for bandits and rebels. In 1872, King George I of Greece bought a

Table 1. Distribution of land uses in Tatoi area.

Land uses	Area, ha
Forest	3930
Cultivated area	88
Bare area	95
Moorland	110
Total area	4223

part of the land, and with subsequent markets he obtained the whole forest. This area was royal property for many years, a fact that upgraded the area. The forest was under management by Danish foresters from 1884–1904, then until 1910 by Germans and then by Greeks. The forest became public three times in 1924, 1973 and 1994, always after a regime change action. Eventually, after lengthy litigations, Tatoi became property of the state in March 2003. Since then, several land uses were phased out, and currently treatments are limited to the removal of dead and decumbent trees. Grazing has been forbidden throughout the area and certain agricultural areas are strictly bordered (Management Plan of Parnitha National Park 1997).

The biggest problem for the forest of Tatoi is fires. This problem is as old as the woods of the region. The prolonged period of dry and hot weather in Attica, in addition to the prevailing flammable vegetation, intense human activity and strong winds, create favourable conditions for causing and rapidly spreading fires. Indicatively, during the period 1913–1996 there were 373 fires, with most incidents occurring July and August. For this period, the burned area reached 985.74 km^2 (Management Plan of Parnitha National Park 1997).

Forest vegetation at higher altitudes of the forest consists mostly of pure stands of *Abies cephalonica* Loud., while at lower

altitudes pure or mixed stands of *Pinus halepensis* Mill. mainly occur, as well as other species such as *Arbutus unedo* L., *Quercus coccifera* L., *Quercus ilex* L., *Phillyrea latifolia* L., *Platanus orientalis* L. and others (Management Plan of Parnitha National Park 1997).

Data collection

Raw data were collected from the entire forest of Tatoi, for the Aleppo pine species (*Pinus halepensis*), in the framework of a Master thesis prepared at the Department of Forestry and Management of the Environment and Natural Resources, of Democritus University of Thrace, in Greece (Angelaki 2013).

Data were collected during the spring of 2012, based on random sampling and GPS use. Coordinates of each Aleppo pine tree were recorded and labelled on a map of the forest. One hundred trees were randomly selected and measured as follows:

- Breast height diameter D (m) was measured with a calliper;
- Total height H (m) was estimated with a Blume-Leiss altimeter;
- Form factor f was estimated with a relaskop.

Total volume V (m³) of each tree was calculated using the formula (2) (Van Laar and Akça 2007):

$$V = \pi \cdot 4^{-1} \cdot D^2 \cdot f \cdot H \quad (2).$$

Regression models

For regression models fitting to data, 32 models were tested (Kitikidou 2008, see Appendix). The first 14 are single-entry models that estimate V in relation to D , while the remaining 18 models are dou-

ble-entry models that estimate V in relation to D and H .

Models' fitting was performed using the statistical software SPSS V.19 (Kitikidou 2005, IBM 2010). Initially, significance of the analysis of variance (Sig. F), of the Variance Inflation Factors – VIFs, of the tolerances and of the t -test for the regression coefficients (Sig. t), were examined. The models that remained for further comparison were those that had: Sig. F < 0.05; VIFs < 10; Tolerances > 0.0001; Sig. t for regression coefficients < 0.05.

Then, the comparison criteria of Table 2 were calculated.

The same criteria used for fitting data were used for validation data. When the selected regression model, based on the values of the above criteria from fitting data, gives similar values for the validation data, then the model is valid, i.e. it will have similar values for the regression coefficients, if we take a new sample (Draper and Smith 1997). The method of separation of the data (split data) mimics the validation of a model by taking a new sample, separating the data that were originally collected in two parts, fitting and validation data (Ezekiel and Fox 1959, Marquardt and Snee 1975). In this study, 80 % of the data (80 trees) were used for fitting and the remaining 20 % (20 trees) for validation.

Results

1. Exploratory data analysis

Before applying statistical approaches aimed at testing specific hypotheses, we applied an Exploratory Data Analysis as a quantitative approach to help understand data. Descriptive statistics of the sampled trees are given in Table 3.

Table 2. Criteria for regression models comparison.

No	Criterion	Formula	Optimum value	References
1	Absolute mean error	$\sum_{i=1}^n \frac{ V_i - \hat{V}_i }{n}$	0	Mayer and Butler (1993) Janssen and Heuberger (1995) Wackerly et al. (2008)
2	Standard error of the estimate	$\sqrt{\frac{\sum_{i=1}^n (V_i - \hat{V}_i)^2}{n-p}}$	min	Ezekiel and Fox (1959) Mathews (1987) Draper and Smith (1997) Wackerly et al. (2008)
3	Coefficient of determination R^2	$1 - \frac{\sum_{i=1}^n (V_i - \hat{V}_i)^2}{\sum_{i=1}^n (V_i - \bar{V})^2}$	1	Ezekiel and Fox (1959) Kvålseth (1985) Mathews (1987) Mayer and Butler (1993) Janssen and Heuberger (1995) Draper and Smith (1997) Everitt and Skrondal (2010)
4	Root of the mean squared error	$\sqrt{\frac{\sum_{i=1}^n (V_i - \hat{V}_i)^2}{n}}$	min	Anderson and Woessner (1992) Draper and Smith (1997)
5	Sum of squared errors	$\sum_{i=1}^n (V_i - \hat{V}_i)^2$	0	Mayer and Butler (1993)
6	Sum of relative squared errors	$\frac{\sum_{i=1}^n \left(\frac{V_i - \hat{V}_i}{V_i} \right)^2}{n} \cdot 100$	0	Draper and Smith (1997)
7	Relative mean squared error, %	$\sum_{i=1}^n \left(\frac{V_i - \hat{V}_i}{V_i} \right)^2$	0	Draper and Smith (1997)
8	Average deviation	$\frac{\sum_{i=1}^n V_i - \hat{V}_i }{\sum_{i=1}^n \hat{V}_i} \cdot 100$	0	Kenney and Keeping (1962) Snee (1977) Havil (2003)
9	Variance ratio	$\frac{\sum_{i=1}^n (\hat{V}_i - \bar{\hat{V}})^2}{\sum_{i=1}^n (V_i - \bar{V})^2}$	1	Mayer and Butler (1993) Janssen and Heuberger (1995) Chen and Deo (2006)
10	Regression coefficients α, β	$V_i = \alpha + \beta \hat{V}_i + \text{error}$	$\alpha=0$ $\beta=1$	Mayer and Butler (1993) Janssen and Heuberger (1995)

Notation: V_i – observed values of volume; \hat{V}_i – estimated values of volume from the regression model; n – number of observations; p – number of regression coefficients; \bar{V} – average of observed volumes; $\bar{\hat{V}}$ – average of estimated volumes.

Boxplots of the measured variables (Figure 1) show a few outliers and some extremes for form factor and volume. All distributions are symmetric, except that the volume appears to be slightly positively skewed.

Table 3. Descriptive statistics of the sampled trees.

Variable	Count	Average	Standard deviation	min	max
<i>D</i> , m	100	0.32	0.09	0.13	0.62
<i>H</i> , m	100	17.42	4.51	8.00	32.00
<i>f</i>	100	0.32	0.08	0.18	0.87
<i>V</i> , m ³	100	0.53	0.47	0.04	2.88

2. Model selection

At first, the problematic models against Sig. *F*, tolerances, VIFs and Sig. *t* for regression coefficients were found and excluded from further comparison. For the remaining models, the comparison criteria of Table 2

were calculated. According to Table 4, the models that had the best values (highlighted) and were selected are the following:

$$V = b_1 \cdot D^2 \cdot H \text{ (model No 15);}$$

$$V = b_0 + b_1 \cdot D^2 \cdot H \text{ (model No 16).}$$

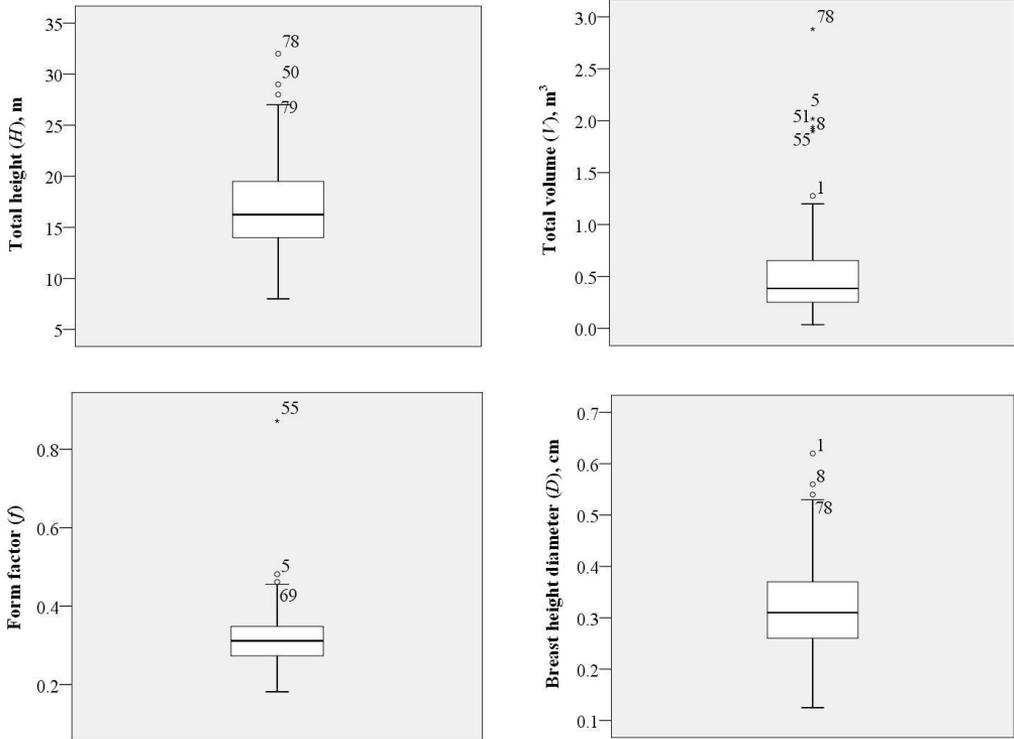


Fig. 1. Boxplots of measured variables.

Table 4. Criteria for regression models comparison – fitting data.

Criterion	Absolute mean error	Standard error of the estimate	Coefficient of determination R^2	Root of the mean squared error	Sum of squared errors	Sum of relative squared errors	Relative mean squared error, %	Average deviation	Variance ratio	Regression coefficients α, β	
Optimum value	0	min	1	min	0	0	0	0	1	0	1
1	0.192	0.299	0.798	0.297	7.041	28.282	35.352	32.486	0.754	-0.142	1.174
9	3046.931	3191.360	<0	3151.215	794412593.7	5215565824.9	6519457281.1	100.018	13476785.51	1.999	0.001
11	3046.931	3191.3560	<0	3151.215	794412593.7	5215565824.9	6519457281.1	100.018	13476785.51	1.999	0.001
15	0.106	0.199	0.910	0.198	3.141	5.132	6.415	18.653	0.894	-0.034	1.037
16	0.101	0.200	0.911	0.197	3.104	4.852	6.066	18.244	0.912	-0.00005	1.000
17	0.144	0.231	0.882	0.227	4.105	121.086	151.358	25.955	0.883	-0.0004	1.000
22	1.501	1.611	<0	1.591	202.514	18966.929	23708.661	158.443	2.780	1.018	0.490
23	1.502	1.607	<0	1.587	201.510	18921.432	23651.790	158.431	2.745	1.027	0.499
24	1.501	1.621	<0	1.590	202.284	19012.550	23765.687	158.470	2.779	1.019	0.491
26	1.501	1.611	<0	1.591	202.514	18966.929	23708.661	158.443	2.780	1.018	0.490

3. Model validation

For the 20 trees used for validation, we followed the same procedure. According to Table 5, comparison of the models that remained after the initial exclusion of models based on Sig. F , tolerances, VIFs and Sig. t for regression coefficients, lead to the selection of the following:

$$V = b_1 \cdot D^2 \text{ (model No 1);}$$

$$V = b_1 \cdot D^2 \cdot H \text{ (model No 15).}$$

4. Final model selection and comparison with the existing one

From the regression models comparison, for the 80 trees (fitting data), we concluded that the best models were nos 15 and 16. For the 20 trees (validation data), the best models were nos 1 and 15. The model that was finally selected and compared with the previous Logging table of the year 2004, was the one that had optimum results in both processes (fitting and validation), i.e. the model no 15.

Table 5. Criteria for regression models comparison – validation data.

Criterion	Absolute mean error	Standard error of the estimate	Coefficient of determination R^2	Root of the mean squared error	Sum of squared errors	Sum of relative squared errors	Relative mean squared error, %	Average deviation	Variance ratio	Regression coefficients α, β	
Optimum value	0	min	1	min	0	0	0	0	1	0	1
1	0.054	0.076	0.988	0.074	0.109	1.798	8.989	12.836	0.996	-0.019	1.037
9	1.439	1.560	<0	1.480	43.780	1287.181	6435.905	141.003	5.946	0.802	0.376
11	1.439	1.560	<0	1.498	43.780	1287.181	6435.905	141.003	5.946	0.802	0.376
15	0.055	0.072	0.989	0.070	0.098	0.716	9.580	13.371	0.980	0.024	0.953
22	1.439	1.564	<0	1.483	44.009	1401.695	7008.473	141.015	5.983	0.789	0.363
23	1.440	1.565	<0	1.485	44.091	1379.696	66898.478	140.970	5.957	0.780	0.354
26	1.439	1.564	<0	1.483	44.009	1401.695	7008.473	141.015	5.983	0.789	0.363

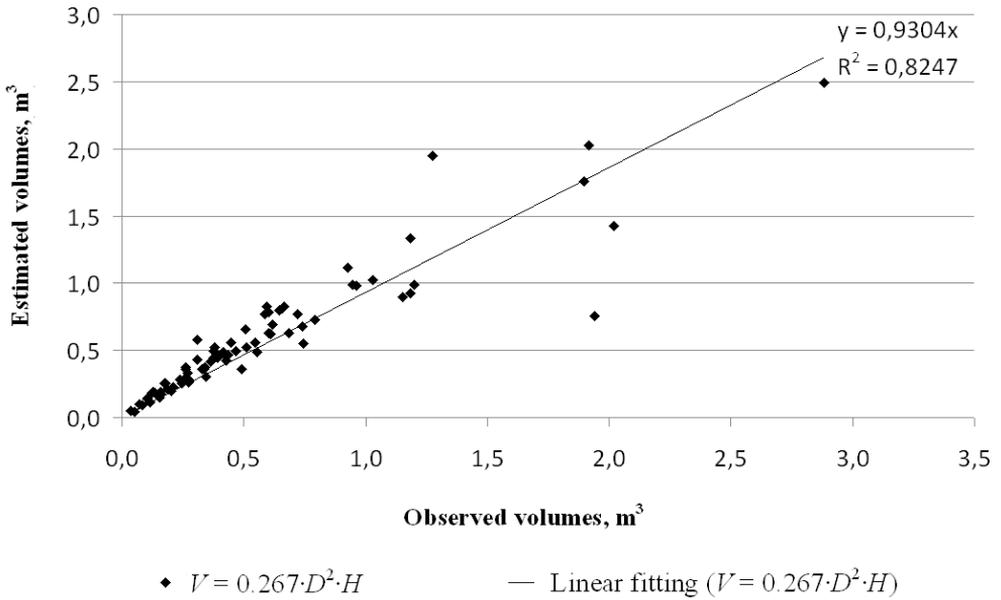


Fig. 2. Linear relationship between observed volumes and estimated from the selected model of the present study.

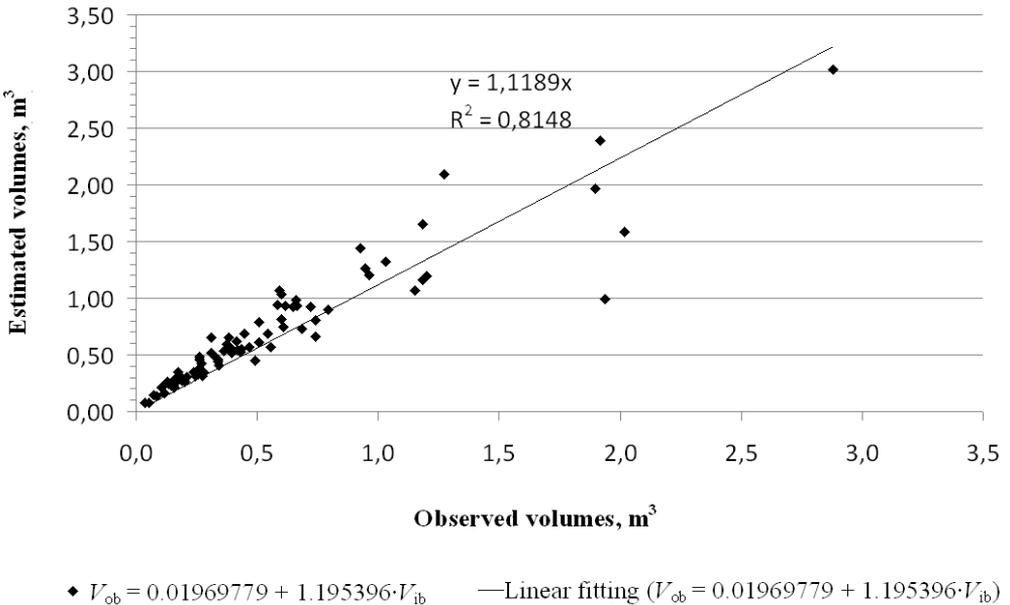


Fig. 3. Linear relationship between observed volumes and estimated from the previous logging table.

Two models were compared:

$V = 0.267 \cdot D^2 \cdot H$ (selection of the present study);

$V_{ob} = 0.01969779 + 1.195396 \cdot V_{ib}$,
where $V_{ib} = 3.3041044 \cdot 10^{-5} \cdot D^{1.790332} \cdot H^{1.181907}$ (selection of the previous logging table).

In the following figures, on the horizontal axis the observed volumes are displayed, while on the vertical the volumes estimated using the model chosen in this study (Figure 2) and the Logging table of 2004 (Figure 3). Ideally, the points of the graphic should form a straight line with a slope of 45°, starting from the origin. One can easily notice that the selected model of this research has a very good fit, like the previous model of the year 2004.

Discussion

There are numerous studies as regards Aleppo pine volume estimation models. Indicatively, we mention Smiris et al. (1998), Montero et al. (2000), Montero et al. (2001), Trasobares et al. (2004), Condés and Sterba (2008). Comparing our model with the one developed by Martínez-Millán et al. (1993) and used in Montero et al. (2001) (mean breast height diameter between inventories in Montero et al. (2001) ranged between 4.39 and 5.46 cm, and mean total tree height between 10.5 and 13.9 m), we can demonstrate that the forest studied in the present research gives slightly smaller volumes per tree. Montero's et al. model is:

$$V = 0.077186 \cdot D^{1.84818} \cdot H^{0.88012},$$

where total volume V is in dm^3 , breast height diameter D in cm and total tree height H in m.

Estimated volume ranges between 0.009 and 0.018 m^3 , while in the forest of Tatoi, the estimated volume, within the same range of breast height diameter and height as Montero et al. (2001), extends between 0.005 and 0.011 m^3 .

With rapidly decreasing forest cover due to human activities, afforestation and reforestation have become a necessity and is being avidly pursued. For Mediterranean, semi-arid and dry climatic zones, *Pinus halepensis* spp. has become a good choice for afforestation and reforestation, due to its drought resistance and good growth potential (Maestre and Cortina 2004). Due to past wildfire cases and from perspective of forest dynamics study, its allometry has recently been emphasized to devise better afforestation and reforestation strategies. Volume and biomass are visual measures of plant's photosynthetic activity and is significant in terms of wildfire control measures, while their patterns give insights into plant physiology and growth (Badía and Martí 2008). Allometric models are the most used tool to assess volume or biomass from forest inventory data (e.g. tree diameter and height) (Picard et al. 2012). Smiris et al. (2000) analyze the aboveground biomass of Aleppo pine forests in Kassandra Peninsula in Chalkidiki in Greece and refer that the knowledge of amount of biomass (flammable material) and its distribution helps in the prevention of forest fires.

In this frame allometry (volume and biomass estimation through diameter and height) should be studied. Prudent volume estimation of *P. halepensis* can greatly enhance success in afforestation and reforestation techniques and better ecological control as well as can help in planning of forest fire prevention.

Conclusion

In this study, a double-entry volume table model was developed and validated, using a random sample of 100 Aleppo pine trees, from the Tatoi forest of Parnitha, Attica. Comparison of the selected model with the pre-existing model of 2004 showed that they are similar, regarding volume estimation. When compared to a study in Spain (Montero et al. 2001), we noticed that the Tatoi forest gives slightly smaller volumes per tree.

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Appendix

Compared regression models

- $V = b_1 \cdot D^2$ (1)
 $V = b_0 + b_1 \cdot D^2$ (2)
 $V = b_0 + b_1 \cdot \pi \cdot 4^{-1} \cdot D^2$ (3)
 $V = b_1 \cdot D + b_2 \cdot D^2$ (4)
 $V = b_0 + b_1 \cdot D + b_2 \cdot D^2$ (5)
 $V = b_0 + b_1 \cdot D + b_2 \cdot D^2 + b_3 \cdot D^3$ (6)
 $V = b_0 + b_1 \cdot D + b_2 \cdot D^2 + b_3 \cdot D^3 + b_4 \cdot D^4$ (7)
 $V = \text{function } [D, D^2, D^{-1}, D^{-2}, \ln D, \ln(D^2)]$ (8)
 $\ln v = b_0 + b_1 \cdot \ln D$ (9)
 $\ln v = b_0 + b_1 \cdot \ln D + b_2 \cdot D^{-1}$ (10)
 $\ln v = \text{function } [D, D^2, D^{-1}, D^{-2}, \ln D, \ln(D^2)]$ (11)
 $v \cdot D^{-2} = b_0 + b_1 \cdot D^{-1}$ (12)
 $v \cdot D^{-2} = b_0 + b_1 \cdot D^{-2}$ (13)
 $v \cdot D^{-2} = b_0 + b_1 \cdot D^{-1} + b_2 \cdot D^{-2}$ (14)
 $V = b_1 \cdot D^2 \cdot H$ (15)
 $V = b_0 + b_1 \cdot D^2 \cdot H$ (16)
 $V = b_0 + b_1 \cdot D^2 + b_2 \cdot H$ (17)
 $V = b_0 + b_1 \cdot D^2 + b_2 \cdot H + b_3 \cdot D^2 \cdot H$ (18)
 $V = b_0 + b_1 \cdot D + b_2 \cdot D^2 + b_3 \cdot D \cdot H + b_4 \cdot D^2 \cdot H$ (19)
- $V = b_0 + b_1 \cdot D^2 + b_2 \cdot H^2 + b_3 \cdot D^2 \cdot H + b_4 \cdot D \cdot H^2$ (20)
 $V = \text{function } [D, H, D^2, H^2, D \cdot H, D^2 \cdot H, D \cdot H^2, D^2 \cdot H^2, D^{-1}, H^{-1}, D^{-2}, (D \cdot H)^{-1}, D^{-2} \cdot H^{-1}, H \cdot D^{-1}, H \cdot D^{-2}, \ln D, (\ln D)^2, \ln H, (\ln H)^2, \ln(D^2), \ln(D^2 \cdot H)]$ (21)
 $\ln v = b_0 + b_1 \cdot \ln(D^2 \cdot H)$ (22)
 $\ln v = b_0 + b_1 \cdot \ln(D \cdot H)$ (23)
 $\ln v = b_0 + b_1 \cdot \ln D + b_2 \cdot \ln H$ (24)
 $\ln v = b_0 + b_1 \cdot \ln D + b_2 \cdot \ln H + b_3 \cdot (\ln D)^2 + b_4 \cdot (\ln H)^2$ (25)
 $\ln v = \text{function } [D, H, D^2, H^2, D \cdot H, D^2 \cdot H, D \cdot H^2, D^2 \cdot H^2, D^{-1}, H^{-1}, D^{-2}, (D \cdot H)^{-1}, D^{-2} \cdot H^{-1}, H \cdot D^{-1}, H \cdot D^{-2}, \ln D, (\ln D)^2, \ln H, (\ln H)^2, \ln(D^2), \ln(D^2 \cdot H)]$ (26)
 $v \cdot D^{-2} \cdot H^{-1} = b_0 + b_1 \cdot D^{-2} \cdot H^{-1}$ (27)
- $\frac{v}{D^2 H} = b_0 + b_1 \cdot D^{-2} + b_2 \cdot H^{-1} + b_3 \cdot D^{-2} \cdot H^{-1}$ (28)
- $\frac{v}{D^2 H} = b_0 + b_1 \cdot (D \cdot H)^{-1} + b_2 \cdot H^{-1} + b_3 \cdot D^{-1} + b_4 \cdot D^{-2} + b_5 \cdot D^{-2} \cdot H^{-1}$ (29)
- $v \cdot D^{-2} \cdot H^{-1} = b_0 + b_1 \cdot H^{-1} + b_2 \cdot (D \cdot H)^{-1} + b_3 \cdot H \cdot D^{-2} + b_4 \cdot D^{-2} \cdot H^{-1}$ (30)
- $v \cdot D^{-2} = b_0 + b_1 \cdot H$ (31)
- $D^2 \cdot H \cdot v^{-1} = b_0 + b_1 \cdot D$ (32)