FORESTRY IDEAS, 2014, vol. 20, No 1 (47): 31–46

DYNAMIC THIRD-GENERATION WHOLE-STAND MODEL FOR SCOTS PINE PLANTATIONS IN BULGARIA

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Received: 26 March 2014 Accepted: 28 April 2014

Abstract

The main objective of this investigation was to derive a dynamic third-generation whole-stand model, which consists of a flexible system of mathematical equations presented graphically – Stand Density Management Diagram (SDMD) – and allows good description of the development of the stands over time. A new SDMD for Scots pine plantations in Bulgaria was elaborated that included two improvements as compared to the preceding SDMD. The Natural thinning curves (NTC) sub-model was presented in a dynamic equation form and was estimated over repeated measurements data from permanent sample plots. The Equivalent height curves (EHC) sub-model was reformulated to include a site index-specific parameter, which produced SDMDs differentiated by site index classes. The principal sub-models of the SDMD were fitted with high level of determination and produced unbiased and statistically significant parameter estimates. The NTC in generalized algebraic difference equation form provided the maximum possible model flexibility by means of stand-specific shape and asymptote of these trajectories. The slope parameter of the self-thinning asymptotes α had value of 1.74. The new model improvements added to its precision and assured improved predictive abilities.

Key words: dynamic growth model, *Pinus sylvestris,* stand density management diagram, Scots pine plantations, stand-specific model parameters, whole-stand model.

Introduction

A model is an abstraction, or a simplified representation, of some aspect of reality and the forest growth model is an abstraction of the natural dynamics of a forest tree or stand, which may encompass growth, mortality, and other changes in stand composition and structure (Vanclay 1994). According to the level of detail they provide, forest growth models are classified into three categories: whole-stand, size-class and individual-tree models. In Bulgaria only whole-stand growth models have been developed and characteristic of this model category is the idea of modeling stand growth by the age development of mean and cumulative stand variables such as height, standing volume or volume growth for use in the assessment, planning and implementation for forestry operations (Pretzsch 2009). The whole-stand growth models are the oldest model type, and are largely represented by the growth and yield tables. Pretzsch (2009) distinguished four generations of whole-stand growth models and the growth and yield tables for normal stands currently implemented in the forest management in Bulgaria can be classified as second generation stand-level models. They present in tabular form the most important stand characteristics according to established silvicultural prescriptions in homogeneous even-aged managed "normal" stands, usually every 5 years.

Besides the traditional for the American and the European forestry practice table form, other whole-stand growth models have been developed and presented for practical application in graphical form. Such models are the Stand Density Management Diagrams (SDMD), which were initially developed in Japan and are primarily used to derive density-control schedules by management objective. The methodology of the SDMD was modified and adapted for application in Bulgaria and was implemented in development of such models for Scots pine plantations in Rila Mountain (Stankova 2004) and for Scots pine and Austrian black pine plantations on national scale (Stankova 2006). These Stand Density Management (or Control) Diagrams are composed of 5 elements, which are defined by principal stand growth parameters (Stankova 2006): Equivalent height curves (EHC), which describe the relationship between stand yield and density at a given growth stage; Full density line, which connects the points of density–maximum yield combinations; Natural thinning curves (NTC), which describe the yield growth of stands of given initial densities with time-lapse, considering the process of self-thinning; Equivalent mean diameter curves (EDC), which connect yield-density combinations of stands having the same mean diameter; and Yield index lines, which are parallel to the Full density line and indicate different stocking levels.

The SDMDs can be subdivided into (1) static, which lack a temporal density decrease sub-model, and (2) dynamic, which include a sub-model for density decrease in time (Newton 2009b). The density decrease sub-model included in the dynamic SDMD usually requires supplementary information about stand age and/or site index (Castedo-Dorado et al. 2009, Newton 2009a, 2012a, 2012b), which is not directly obtainable from the diagram. Another approach in modeling the dynamic component of the SDMD is to express directly volume or biomass growth in time as a function of the decrease in density (Tadaki 1964, Shibuya et al. 1997, Stankova and Shibuya 2007) in a set of natural thinning trajectories, thus bypassing the sub-model of density decrease with time. In spite of the intrinsically dynamic nature of the SDMD developed in Bulgaria, their dynamic sub-model – the Natural thinning curves – was not estimated over repeated measurements data from permanent sample plots, but was indirectly determined from the parameters of the full-density line (Stankova 2006, Stankova and Shibuya 2007); this sub-model showed potential for precision improvement (Stankova 2007). An investigation on the accuracy of the Equivalent height curves through validation with experimental data sets suggested possibility to improve their predictive abilities through SDMD differentiation by site index classes (Stankova and Petrin 2008).

The main objectives of the present investigation are: 1) to reformulate the

NTC sub-model of the SDMD in dynamic equation form; 2) to derive site index-specific EHC sub-model, and 3) to apply the improvements in the elaboration of a new dynamic SDMD model for Scots pine (*Pinus sylvestris* L.) plantations in Bulgaria.

Materials and Methods

Data set

The data set was generated from both personally collected and published data records. The first source of data comprised 115 variable-sized (of circular or rectangular form) temporary sample plots established and measured in 2002–2005. In each plot, breast height diameters of all trees, samples of average and dominant tree heights and the number of trees were recorded. Mean and dominant stand heights, stand basal area and stand density, expressed as number of trees per hectare, were calculated from the plot measurements. The mean and the total stand volumes were calculated from the individual stem volumes of the trees within each plot, which were determined through the volume tables for Scots pine plantations by Krastanov et al. (1983). Beside the personally recorded data, additional data from 340 plots in *P. sylvetsris* plantations either published or granted by other researchers were included in the data set. The sources of these data are permanent and temporary sample plots data published in PhD theses, MSc thesis, books, articles, and research reports (Stankova 2006, Appendix 2). Data records from 21 permanent sample plots installed in 1–3 replications and re-measured 1 to 3 times were obtained from Forest Inventory Plans. In addition, 46 measurement sets used for developing growth and yield tables for Scots pine plantations in Bulgaria (Krastanov et al. 1980) were also included as part of the parameterization data set. Dominant heights for the published data were additionally determined using the established allometric relationship to the mean height for Scots pine plantations (Stankova et al. 2006). Data sub-sets that contained the dependent and all independent variables, required by the respective sub-model, were drawn from the total data set to parameterize each particular principal sub-model of the SDMD. Both longitudinal and cross-sectional data were used to fit the EDC sub-model and the EHC sub-model. Dominant heights were grouped into 1 m classes for the EHC sub-model. Site index was estimated by plots for reference age 50 years, according to the dynamic site index model by Stankova and Diéguez-Aranda (2012) and data were classified into four-meter site index classes. Three of them – 16, 20 and 24 m – presented sufficient amount of data for parameterization of the EHC sub-model, which was assessed through a preliminary screening. Data within each site index class were examined by height classes and only those height classes, which had at least 3 measurements and revealed adequate estimation of the reciprocal relationship between density and mean volume (Stankova 2006), with 2 positive model parameters, were included in the parameterization data set. The dominant height classes, which were finally selected, spanned uniformly the density mean volume data for each of site index classes 16, 20 and 24 m (Table 1). The estimation of dynamic NTC requires longitudinal data and measurements from 48 permanent plots were available for it. To overcome the scarce amount of data for this sub-model, 81 more plots (initial density sub-set) were added. Two measurements in time were available for each plot of the latter sub-set, the first-one representing the initial planting density coupled with initial mean volume value, which was set equal to zero. Summary of the parameterization data sets by sub-models is shown in Table 1.

Table. 1. Descriptive statistics of the data sets used to parameterize the principal sub-models of the dynamic SDMD.

SDMD Sub-model	Stand variable**	Mean	Standard Deviation	Minimum	Maximum
Equivalent Mean Diameter Curves (*n*=609)	Quadratic mean diameter, cm	14.8	6.9	2.2	38.0
	Dominant stand height, m	14.7	6.1	1.5	32.5
	Stand density, ha^{-1}	2986	2451	360	20000
	Total stand volume, $m^3 \cdot ha^{-1}$	295.1	174.9	2.2	909.7
Equivalent Dominant Height Curves	**Site index 16m (*n*=57)**				
	Dominant height class, m	11	4	5	15
	Stand density, ha^{-1}	4152	2888	1505	20000
	Mean stem volume, m^3	0.0710	0.0499	0.0029	0.1936
	Site index 20m (*n*=204)				
	Dominant height class, m	13	6	4	27
	Stand density, ha^{-1}	3597	2544	466	16800
	Mean stem volume, m^3	0.1308	0.1613	0.0018	1.1578
	Site index 24m (*n*=90)				
	Dominant height class, m	16	5	6	22
	Stand density, ha^{-1}	2925	2300	553	12550
	Mean stem volume, m^3	0.1632	0.1295	0.0056	0.4884
Natural Thinning Curves*	**Permanent Sample Plots (*n*=133)**				
	Stand density, ha^{-1}	2375	2384	393	20000
	Mean stem volume, m^3	0.2893	0.2488	0.0029	1.2568
	Initial density sub-set (*n*=162)				
	Stand density, ha^{-1}	4827	3187	1723	20000
	Mean stem volume, m^3	0.0353	0.0513	0	0.2441

Note: * – ***n*** for the NTC sub-model denotes plot – measurement occasion combinations;
** – the dependent and independent variables of each sub-model are included.

Dynamic SDMD model derivation

The new dynamic SDMD for Scots pine plantations provides improvements to the preceding model (Stankova 2006, Stankova and Shibuya 2007) in two ways. First, the NTC sub-model was reformulated from integral to generalized algebraic difference equation form, thus deriving a dynamic equation to be estimated over a data set from repeated measurements of permanent sample plots. Next, the EHC sub-model was reformulated to include a site index-specific parameter, providing in this manner different SDMD model for each site index class.

The integral equation form of the NTC sub-model is suggested by Shibuya (1995) and presents the mean stem volume as a function of the number of trees per hectare:

$$v = K \cdot N^{-\alpha} - f \qquad (1),$$

where: v is the mean stem volume (m^3), N is stand density (ha^{-1}), and α, K and f are parameters. Parameter f by definition is stand-specific and depends on the initial stand density, while parameters K and α correspond to the intercept and the slope of the full-density line (Shibuya 1995). The latter two parameters are often considered species-specific, i.e. global model parameters, but some studies suggest that the intercept K is not a global, but a specific parameter, which depends not only on the tree species, but also on site quality, species mixture and stand origin (Shibuya 1995, Weiskittel et al. 2009). In this study, we assumed that α is global, while K is site-specific. At the initial stand conditions, mean stem volume approaches zero, while N obtains the initial stand density value N_0:

$$K \cdot N_0^{-\alpha} - f = 0 \Rightarrow N_0^{\alpha} = \frac{K}{f} \sim f(X) \qquad (2)$$

Equation 2 suggests that, if there is a complex unobservable independent variable X, which combines the effect of variables otherwise not accounted for in the model (e.g. initial stocking rate, soil conditions, ecological and climatic factors) and which is incorporated in the natural thinning model, the two specific model parameters K and f should be related to it in an inverse manner. Thus, the following reciprocal formulations are suggested:

$$K = p \cdot X,\ f = \frac{1}{X} \qquad (3),$$

where p is a global model parameter. By substitution of equations 3 into equation 1 and after solving for the initial conditions N=N_1 and v=v_1 the following expression for X is obtained:

$$X = \frac{v_1 + \sqrt{v_1^2 + 4 \cdot p \cdot N_1^{-\alpha}}}{2 \cdot p \cdot N_1^{-\alpha}} \qquad (4)$$

After substitution of equation 4 into equation 1 for the current conditions N=N_2 and v=v_2, the Generalized Algebraic Difference Approach (GADA) equation is derived:

$$v_2 = p \cdot X \cdot N_2^{-\alpha} - \frac{1}{X} = \frac{N_1^{\alpha} \cdot N_2^{-\alpha}}{2}\left(v_1 + \sqrt{v_1^2 + 4 \cdot p \cdot N_1^{-\alpha}}\right) - \frac{2 \cdot p \cdot N_1^{-a}}{v_1 + \sqrt{v_1^2 + 4 \cdot p \cdot N_1^{-\alpha}}} \qquad (5)$$

Equation 5 provides prediction for the mean stem volume, based on the stand density value (N_2) and the past records for the mean stem volume and stand density (N_1, v_1).

The EHC sub-model is based on the reciprocal equation by Hagihara (2000):

$$v = \frac{1}{At \cdot N + B} \quad \text{or} \quad V = \frac{N}{At \cdot N + B} \quad (6),$$

where: V is the total volume per hectare ($m^3 \cdot ha^{-1}$), while At and B are model parameters that are functions of time, but independent of density:

$$At = \frac{1}{v_0 \cdot N_i^*}\left(e^{-(\alpha-1)\mu \cdot \tau} - e^{-(1-\mu)\tau}\right) \quad (7)$$

$$B = \frac{e^{-\tau}}{v_0} \quad (8),$$

where: v_0 is a constant irrespective of the initial density and is equal to the initial mean stem volume, τ is called biological time (an integrated value of the coefficient of growth $\lambda(\tau)$ with respect to physical time t), μ is a specific constant corresponding to the relative mortality rate as τ tends to be finitely large, N_i^* is the initial density of a population that obeys the 3/2 power law of self-thinning from the start of the experiment (Hagihara 2000). Equation 8 can be reformulated as:

$$e^{-\tau} = B \cdot v_0 \quad (9)$$

Substitution of equation 9 into equation 7 allows the following formulation of At as a function of B:

$$At = \frac{1}{v_0 \cdot N_i^*}\left[(B \cdot v_0)^{\mu(\alpha-1)} - (B \cdot v_0)^{1-\mu}\right] =$$

$$= \frac{v_0^{\mu(\alpha-1)}}{v_0 \cdot N_i^*} B^{\mu(\alpha-1)} - \frac{v_0^{1-\mu}}{v_0 \cdot N_i^*} B^{1-\mu}$$

$$\Updownarrow$$

$$At = c_1 \cdot B^{d_1} - c_2 \cdot B^{d_2} \quad (10),$$

where:

$$c_1 = \frac{v_0^{\mu(\alpha-1)}}{v_0 \cdot N_i^*} \quad (11)$$

$$c_2 = \frac{v_0^{1-\mu}}{v_0 \cdot N_i^*} \quad (12)$$

$$d_1 = \mu(\alpha - 1) \quad (13)$$

$$d_2 = 1 - \mu \quad (14)$$

Next, the representation of parameter B as a power function of the dominant height class ($\hat{H}$, m) was considered (Stankova and Shibuya 2007):

$$B = a_2 \cdot \hat{H}^{-b_2} \quad (15),$$

where: a_2, b_2 are model parameters. The output of a preliminary fit of the reciprocal equation 6 over the experimental data, i.e. the estimates for parameter B through equation 6, was used to examine the power relationship of equation 15. It was fitted once setting b_2 global and a_2 site index specific parameter and then setting a_2 global and b_2 site index specific parameter. The results showed that parameter a_2 was not significant when fitted as specific parameter, while parameter b_2

varied significantly by site index classes. Consequently, parameter a_2 was derived as global, while parameter b_2 – as specific model parameter.

After substitution of equation 10 into equation 6, the EHC sub-model was expressed as a function of two independent variables (dominant height class and number of trees per hectare), five global (c_1, c_2, d_1, d_2 and a_2) and one (b_2) site index specific parameters:

$$v = \frac{1}{N(c_1 \cdot B^{d_1} - c_2 \cdot B^{d_2}) + B},$$

where $B = a_2 \cdot \hat{H}^{-b_2}$ (16)

The other sub-models of the dynamic SDMD followed the formulations suggested in the previous studies (Stankova 2006, Stankova and Shibuya 2007):

Equivalent mean diameter curves sub-model:

$$V = \frac{\pi \cdot N \cdot QMD^2(a \cdot H + c)}{40000} \quad (17)$$

Full density line:

$$v_{FDL} = K \cdot N^{-\alpha} \quad (18)$$

Yield index lines:

$$v_{YIL} = YI \cdot v_{FDL} \quad (19),$$

where: H is dominant stand height (m), QMD is stand quadratic mean diameter, a and c are global model parameters in an expression that derives average stand form height as a linear function of dominant stand height (equation 17), YI denotes yield index that corresponds to the stand stocking rate and obtains values between 0 and 1, v_{FDL} denotes the mean stem volume of stands on the full density line, while v_{YIL} denotes the mean stem volume of stands of yield index YI.

Estimation procedure and goodness of fit statistics

The estimation procedure required application of a sequence of several steps for model parameterization. First, the dynamic NTC sub-model was fitted in algebraic difference equation form (equation 5), using a similar method to the Base-Age Invariant (BAI) dummy variable approach (Cieszewski et al. 2000), that exhibits invariance property of the parameter estimates. The BAI methods are proposed and developed where dominant height is the dependent variable and stand age is the independent variable, but was adapted here for a model formulation in which stand density is the independent variable and mean stem volume is the dependent variable; therefore, it is a “Base-Density Invariant (BDI)” method. The mean stem volume corresponding to the base density was estimated as plot specific parameter employing dummy variables p_i of value 1 for plot i and 0 otherwise. Next, the value of the global slope parameter α estimated through equation 5 was used to constrain the value of parameter d_1 against d_2 in the EHC sub-model (equation 16). If parameter μ in equation 14 is expressed by parameter d_2 and is further substituted into equation 13, the following relationship is derived:

$$d_1 = \alpha - 1 - (\alpha - 1)d_2 \quad (20)$$

Consequently, the estimated value for α allows reduction in the number of parameters of equation 16 through implementation of equation 20 as a constraint. Equations 11–14, on the other hand, propose the following formulation for the constant v_0 (corresponding to the initial mean stem volume at the time of stand establishment):

$$v_0 = \left(\frac{c_1}{c_2}\right)^{\frac{1}{d_1 - d_2}} \qquad (21)$$

The values of the model parameters c_1, c_2, d_1, d_2 can be constrained in such a way that v_0 attains practically meaningful value by employing an appropriate empirical constant, e.g.

$$v_0 = \left(\frac{c_1}{c_2}\right)^{\frac{1}{d_1 - d_2}} < 0.0005,\ m^3 \qquad (22)$$

The constant 0.0005 m^3 was obtained from the volume tables for Scots pine plantations (Krastanov et al. 1980) and corresponds to the mean stem volume of a tree of 1cm diameter at breast height from the lowest height classes (12–15 m). Parameter b_2 was expanded to include dummy variables (q_i) and parameters associated with the particular site indices (b_{2i}), i.e.

$$b_2 = \sum_{i=16,20,24} q_i \cdot b_{2i} \qquad (23),$$

where q_i are the dummy variables of value 1 for site index i and 0 otherwise. After fitting the EHC sub-model (equation 16) with simultaneous application of the derived constraints (equations 20 and 22), the initial mean stem volume at the time of stand establishment v_0 was estimated from equation 21 and subsequently substituted in the initial density data subset for the NTC sub-model (Table 1). Then, the NTC sub-model was refitted over the updated data set, which did not change the slope parameter value α to the second decimal place, but provided more precise estimation for parameter p (equation 5). The GADA form of the NTC sub-model suggests stand-specific shape (f) and intercept (K) of the estimated trajectories, thus deriving set of polymorphic curves with multiple asymptotes (Figure 1). The uppermost of the resultant asymptotes was defined as the Full density line, i.e. $v_{FDL} = K_{FDL} \cdot N^{-\alpha}$, $K_{FDL} = \max(p \cdot X)$. The elaboration of the dynamic SDMD was completed with the estimation of the EDC sub-model through equation 17.

The three principal sub-models of the SDMD (EHC, EDC and NTC) were fitted through nonlinear least squares method, applying the Marquardt estimation algorithm. The combined time-series cross-sectional nature of the remeasurement data used to fit the EHC and the EDC contributed to manifestation of heterogeneous variances (heteroscedasticity), which was detected through graphical and analytical tests, and application of Heteroscedasticity-Consistent Covariance Matrix Estimation (HCCME) (Long and Ervin 2000) was applied to assure the efficiency of the regression estimates for these two sub-models. Furthermore, the assumption of independent errors is very likely to be violated in estimating dynamic regression equations with data from repeated measurements of permanent sample plots (e.g. the NTC). However, the graphical and analytical tests for presence of heteroscedastic errors did not provide evidence for violation of this regression requirement regarding the data set for estimation of the NTC. Tests for the presence of serial correlation, on the other hand, might be inadequate, because forest stand data do not usually constitute a single series (e.g., the present study), but rather a multiplicity of concurrent, relatively short time series (*sensu* Gregoire 1987). The lack of indisputable evidence for violated requirements to the residuals, together with the earlier-mentioned scarcity of the parameterization data for this sub-model, prevented us from application of any specific corrections while fitting this regression model (equation 5).

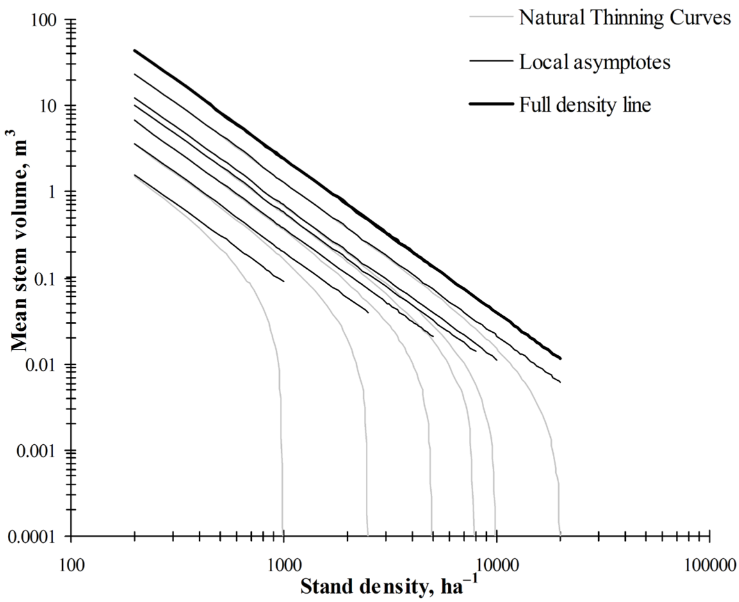

Fig. 1. Natural Thinning Curves sub-model.

The goodness-of-fit of the principal sub-models was assessed on the basis of the adjusted coefficient of determination (R^2_{adj}), the Root Mean Square Error (RMSE), the *Bias* and the %*Bias*:

$$R^2_{\mathrm{adj}} = 1 - \frac{(n-1)\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{(n-k)\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2} \quad (24)$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{n}} \quad (25)$$

$$Bias = \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)}{n} \quad (26)$$

$$\%Bias = \frac{\bar{y} - \bar{\hat{y}}}{\bar{y}}100 \quad (27),$$

where: y_i and $\hat{y}_i$ are observed and predicted values of the dependent variable (mean stem volume *v* for NTC and EHC and total stem volume *V* for EDC) on the i^{th} plot, *n* is the sample size, *k* is the number of model parameters, $\bar{y}$ is the mean observed and $\bar{\hat{y}}$ is the mean predicted value of the dependent variable.

We further assessed the predictive abilities of the models by considering the 95% confidence (*CI*), prediction (*PI*) and tolerance (*TI*) error intervals:

$$95\%\ CI = Bias \pm \frac{S \cdot t_{0.975}}{\sqrt{n}} \quad (28)$$

$$95\% PI = Bias \pm S \cdot t_{0.975}\sqrt{\left(1+\frac{1}{n}\right)} \quad (29)$$

$$95\%\ TI = Bias \pm S \cdot g\left(1-\gamma, n, 1-\alpha\right)$$

for $1-\gamma = 1-\alpha = 0.95$ (30),

where: *S* is the standard deviation of the errors, $t_{0.975}$ is the 0.975 quantile of the *t* distribution with *n*–1 degrees of freedom, the function g (1–γ, *n*, 1–α) is the tolerance factor tabulated for specified values of *n,* α and γ and provides that the estimated interval will contain at least (1–γ) 100 percent of the future error distribution with probability (1–α). The model bias was additionally assessed by linear regressions of observed against predicted values of the dependent variable and simultaneous *F*-test for line slope equals 1 and zero intercept (Gadow and Hui 1999).

Results

The principal sub-models of the SDMD were fitted with high level of determination (R^2_{adj} above 0.9 for all three sub-models) and produced unbiased and statistically significant parameter estimates (Table 2). The %*Bias*, which indicates by how much the mean of the predictions deviates from the observed mean, obtained values of less than 1 % for all sub-models. The plots of the predictions against the observations, supported by the analytical simultaneous *F*-test, which examines the null hypothesis that the regression line between them has slope equal to 1 and intercept equal to zero, confirmed the goodness-of-fit of the three principal sub-models (Figure 2).

Three sets of EHC were estimated, each of them corresponding to a different site index class (Figure 3) and designated by differing value of the exponent for parameter B (equation 23), which obtained decreasing and statistically significant estimates (Table 2). The slope parameter of the self-thinning asymptotes α had value of 1.74, while the intercept of the Full density line was K = 333867.193.

Discussion

Considering the classification by Pretzsch (2009), the Stand Density Management Diagrams of this study can be categorized as a third generation whole-stand model. Pretzsch (2009) defines that the core element of the third generation stand-level models consists of a biometric model in the form of a flexible system of mathematical equations that are normally transferred to computer programs and predict stand development in relation to a specified management regime, site index and yield level. Although presented and utilized in the form of complex charts, which carry information on the spatial-temporal dynamics of the principal stand-level growth and yield attributes, the SDMD are based on a system of mathematical relationships

Table 2. Goodness-of-fit estimates for the principal sub-models of the dynamic SDMD.

Sub-model	Model fit				Parameter estimation					Error intervals					
	R^2_{adj}	[a] RMSE	[b] Bias	Percent bias, %	Global parameter estimate			Standard error	*t*-value	Confidence interval		Prediction interval		Tolerance interval	
NTC	0.950	0.0475	0.0003	0.2322	α		1.7394	0.1018	17.092***	–0.0034	0.0048	–0.0693	0.0708	–0.0742	0.0756
					p		3347.6	1518.7	2.2043*						
EHC	0.938	0.0358	–0.0011	0.8713	c_1		0.0346	0.0024	14.585***	–0.0023	0.0051	–0.0686	0.0714	–0.0730	0.0758
					c_2		0.0335	0.0025	13.159***						
					d_1		0.4245	3.20×10^{-7}	1.33×10^{6}***						
					d_2		0.4263	3.09×10^{-7}	1.38×10^{6}***						
					a_2		24166	1.28×10^{-5}	1.88×10^{9}***						
					b_2	*SI* 16	3.3122	0.1028	32.228***						
						SI 20	3.2950	0.1064	30.964***						
						SI 24	3.2025	0.0620	51.630***						
EDC	0.985	21.116	–0.2930	–0.0568	a		0.4082	0.0050	80.8692***	–2.52	0.84	–42.33	40.64	–44.30	42.61
					c		1.6764	0.0866	19.3472***						

Note: * – Significant at $P < 0.05$; *** – Significant at $P < 0.001$; [a] – RMSE and Bias for NTC and EHC are measured in m^3; while for the EDC in $m^3 \cdot ha^{-1}$; [b] – the absolute biases are not significantly different from zero.

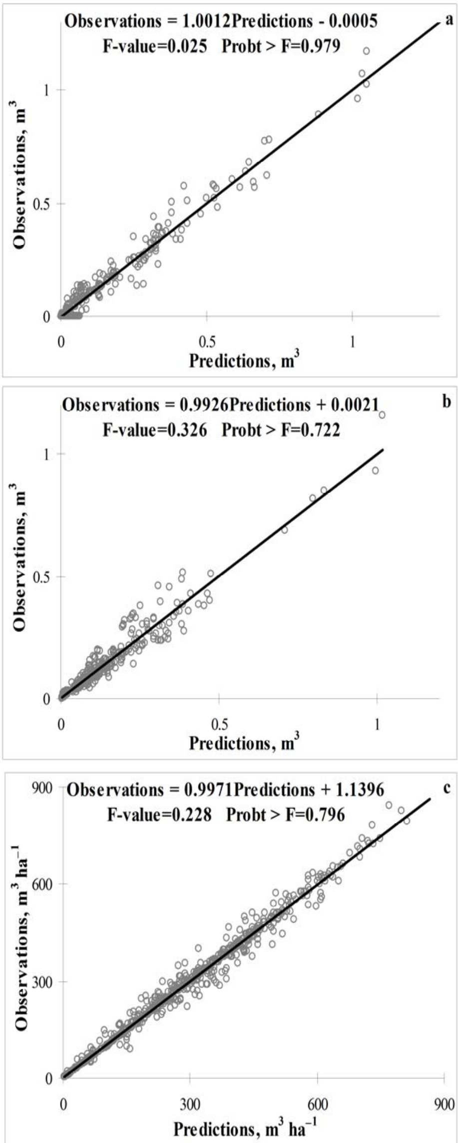

Fig. 2. Plot of observed against predicted values for the principal sub-models: a) NTC; b) EHC; c) EDC.

Note: Linear regressions of observations against predictions are fitted and the results from simultaneous *F*-test for line slope equals 1 and zero intercept (Gadow and Hui 1999) are shown in the plots.

(Stankova and Shibuya 2007, Castedo-Dorado et al. 2009, Newton 2009a), which allow also model incorporation and its further utilization through a computer program (Newton 2012a, 2012b).

García (1988) defined two principal types of growth models: static and dynamic growth models. The static models attempt to predict directly the course over time of the quantities of interest (volumes, mean diameter) and can give good results for unthinned stands, or for stands subject to a limited range of standardized treatments (García 1988). The earlier version of the SDMD for Scots pine plantations (Stankova 2006, Stankova and Shibuya 2007) appeared to be static third-generation whole-stand model, because its density decrease sub-model in time was estimated in integral equation form. Dynamic growth models, on the other hand, predict the rates of change under various conditions, with the time trajectories obtained by adding or integrating these rates. They are needed for forecasting over a wider range of tending regimes and provide a better description of the development of the stand over time than static models (García 1988). To make use of the advantages of the dynamic growth models over the static, a dynamic NTC sub-model was developed in this study. The generalized algebraic difference equation form derived here provides the maximum possible flexibility of this set of curves, because allows stand-specific shape and asymptote of the natural thinning trajectories.

The NTC sub-model expresses directly volume growth in time as a function of the decrease in density (equations 1 and 5) and bypasses the density-decrease sub-model. Such approach for growth projection in time, however, does not include the time variable, expressed through age or

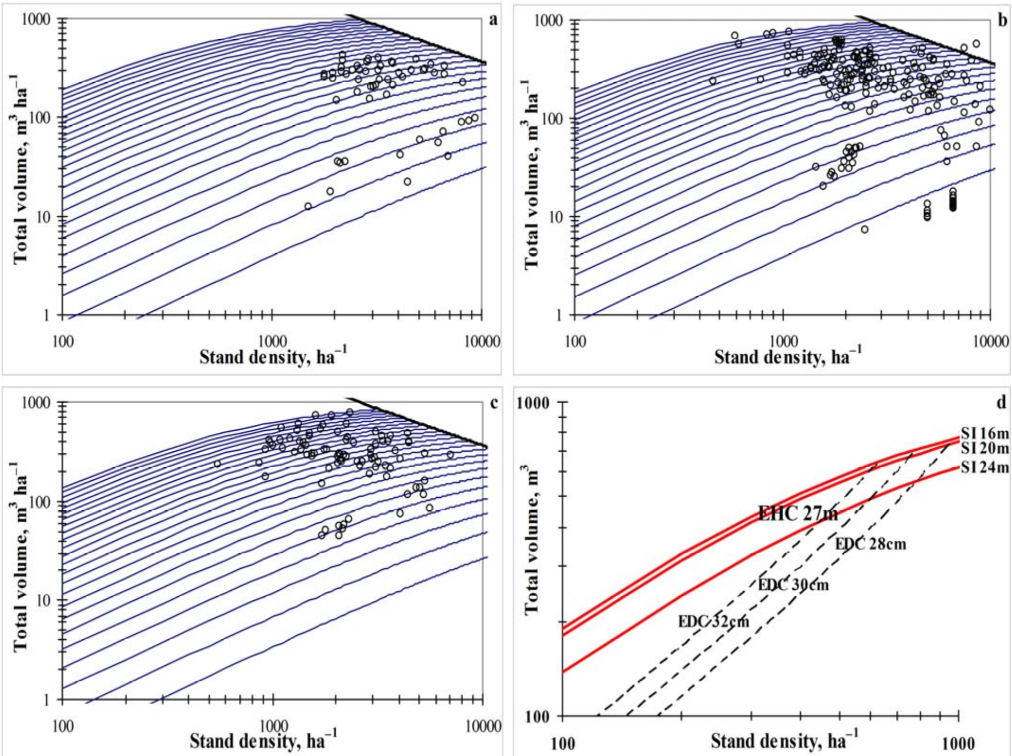

Fig. 3. Equivalent Dominant Height Curves differentiated by site index: a) Site index 16m; b) Site index 20 m; c) Site index 24 m; d) Comparison of EHC for dominant height class 27 m by Site indices.

Note: The parameterization data sets are indicated by circles, Equivalent Height Curves (EHC) by solid lines and Equivalent Diameter Curves (EDC) by dotted lines.

some measure of tree size that is itself a function of time. Thus, if no density decrease occurs in the stand, which can be characteristic for the first (before the onset of the competition-induced mortality) and the last (the phase of growth-resources equilibrium) phases of stand growth, no alteration in the tree size with the progress of time can be predicted. This is not an issue for the Scots pine plantations, which are characterized by relatively high initial densities (over 1000 seedlings per hectare), where competition-induced mortality commences soon after plantation establishment. However, in case of intensively grown plantations of low initial densities, which are managed at short rotation periods and practically do not undergo density-dependent mortality during their life span, this growth model would be inapplicable.

The EHC sub-model of the present study was differentiated by site indices by specifying one of the model parameters as site index-specific. Table 2 shows that the exponent b_2 obtains decreasing with the site index class values and the difference is particularly pronounced for site index 24 m. Considering equation 16, this parameter trend suggests that mean and total stem volume will obtain lower values for higher site index for the same values of dominant height class and stand density, which is seen also in Figure 3. The same dominant height is attained at earlier age by more productive (high site index) stands than by stands of low growth potential from low quality forest sites. Consequently, the more advanced age for attainment of the same dominant height by stands of lower site index can be viewed as related to bigger overall stem size, moreover the temporal trends of height and diameter differ. This explanation is supported by the assertion of Weck (sensu Assmann 1970) that a given stand height is accompanied by a given total increment which is higher the longer it takes the stand to reach this given height. In agreement with this statement, the total crop volume for a particular height (e.g. dominant stand height 27 m, Figure 3d) is expected to be larger if this height is reached at a later age, i.e. for the lower site quality classes (Figure 3d). Also, dominant height increment is insignificantly influenced by the process of thinning (natural or man-made) and consequently reflects merely the height growth trend of the dominant trees. The stand mean diameter growth, on the other hand, is a result of both the tree diameter growth and the mortality of thinner and weaker trees, which artificially inflates the value of the mean stand diameter. Thus, the mean stand diameter can be expected to have steeper growth gradient than the stand dominant height. The differing mean stand diameter values as a possible explanation of the observed distinctions between the volumes of the EHC of different site indices is also easily derivable from the EDC sub-model (equation 17) and is well illustrated on Figure 3d. Indeed, the quadratic mean diameter estimated for stands of the same dominant height class and stand density, but having lower site index value (16 or 20 m) exceeds by about 2 cm the mean diameter of the stands of higher site index value (Figure 3d). This finding agrees with the conclusion by Assmann (1970) that even in geographically small regions, a uniform relationship of total volume on stand height cannot be assumed and the large observed differences in the yield level are determined by differences in the basal area increment and the local basal area potential of the stands for a given mean height.

Conclusions

A new SDMD for Scots pine plantations was elaborated, which presents a dynamic third-generation whole-stand growth model. The NTC sub-model was reformulated into dynamic equation form and was estimated over repeated measurements data from permanent sample plots, which allowed better description of the development of the stands over time as compared to the former static SDMD. The EHC sub-model was reformulated to include a site index-specific parameter, which produced SDMDs differentiated by site index classes that added to the model precision and assured its improved predictability. The newly elaborated dynamic SDMD is applicable for estimation of the stand-level growth parameters at any stand growth stage and for simulation of alternative thinning regimes in accordance with a preferred management objective, with projection of the future stand growth and evaluation of the intermediate and total harvest. Incorporation of the model into software is planned to facilitate its implementation as a management tool.

Acknowledgements

We are grateful to the Forest Research Institute of the Bulgarian Academy of Sciences, who made available, through the research reports and theses of its possession, some of the data from Scots pine plantations used in this study. This work was financially supported by the Marie Curie Intra-European Fellowship Project within the 7th European Community Framework Programme: "Elaboration of advanced-level models for density management of coniferous and broadleaved even-aged natural stands and plantations in Europe" (PIEF-GA-2009-235039), conducted at the University of Santiago de Compostela, Spain.

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